1 Equipment

Included:

1	Resistor/Capacitor/Inductor Network	UI-5210
1	Voltage Sensor	UI-5100
1	Short Patch Cord Set	SE-7123
1	850 Universal Interface	UI-5000
1	PASCO Capstone	UI-5400

2 Introduction

The manner by which the voltage on a capacitor decreases is studied. The half-life for the decay is measured directly and also calculated using the capacitive time constant. This experiment requires a formal Lab Report as described in the "Lab Report Format" document.

3 Theory

Capacitors are circuit devices that can store charge. The capacitance (size) of the capacitor is a measure of how much charge it can hold for a given voltage.

$$Q = CV_C \qquad (1)$$

where C is the capacitance in Farads, Q is the charge in Coulombs, and V_C is the voltage across the capacitor in Volts.

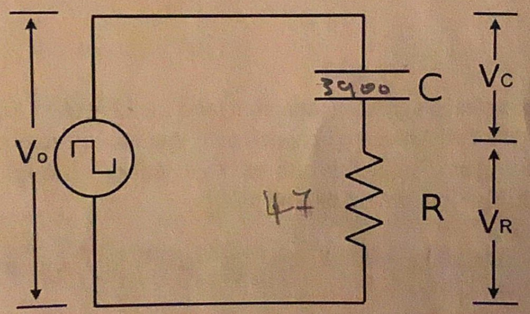

Figure 1. RC Circuit Voltages

To determine how the charge on a capacitor decays in time, use Kirchhoff's Loop Rule for Figure

1:

$$V_o = V_C + V_R \qquad (2)$$

Solving Equation (1) for the voltage across the capacitor gives

$$V_C = Q/C \qquad (3)$$

The voltage across the resistor is given by Ohm's Law:

$$V_R = IR \qquad (4)$$

Therefore,

$$V_o = Q/C + IR \qquad (5)$$

Since the applied voltage, V_0, is zero when the capacitor is discharging, Equation (5) reduces to

$$Q/C + IR = 0 \qquad (6)$$

Since the current is

$$I = \frac{dQ}{dt} \qquad (7)$$

Equation (6) becomes the differential equation

$$\frac{dQ}{dt} + \left(\frac{1}{RC}\right)Q = 0 \qquad (8)$$

Solving Equation (8) for Q gives

$$Q = Q_{max}e^{-\left(\frac{t}{RC}\right)} \qquad (9)$$

Plugging Q into Equation (2) gives the voltage across the capacitor as a function of time

$$V(t) = V_o e^{-\left(\frac{t}{RC}\right)} \qquad (10)$$

where $V_o = Q_{max}/C$. The rate that voltage across a capacitor (and the charge stored in the capacitor) decreases depends on the resistance and capacitance that are in the circuit. If a capacitor is charged to an initial voltage, V_o, and is allowed to discharge through a resistor, R, the voltage, V, across the capacitor will decrease exponentially.

The half-life, $t_{1/2}$ is defined to be the time that it takes for the voltage to decrease by half:

$$V(t_{½}) = V_o/2 = V_o e^{-t_{½}/RC} \qquad (11)$$

Solving for the half-life gives $t_{½} = RC \ln 2$. (12)

The product RC is called the capacitive time constant and has the units of seconds.

4 Pre-lab Questions

1. Show that the capacitive time constant RC has units of seconds.
2. If the capacitance in the circuit is doubled, how is the half-life affected?
3. If the resistance in the circuit is doubled, how is the half-life affected?
4. If the charging voltage in the circuit is doubled, how is the half-life affected?
5. To plot the equation $V(t) = V_o e^{-t/RC}$ so the graph results in a straight line, what quantity do you have to plot vs. time? What is the expression for the slope of this straight line?

5 Setup

1. Construct the circuit shown in Figure 2. The voltage source is Signal Generator #1 on the 850 Universal Interface. C = 3900 pF (capacitor C1 on the UI-5210 board) and R = 47 kΩ (resistor R1 on the UI-5210 board).

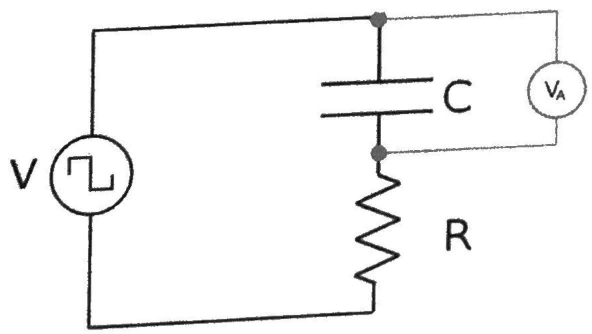

Figure 2. RC Circuit Diagram

2. Click on Signal Generator #1 to connect the internal Output Voltage-Current Sensor. Set the signal generator to a 350 Hz square wave with 2 V amplitude and 2 V offset. This will make the square wave all positive with an amplitude of 4 V. Set the signal generator on Auto.

3. Plug the Voltage Sensor into Channel A. Connect the Voltage Sensor across the capacitor.

6 Procedure

1. Set up an oscilloscope display with the Voltage Ch. A and the Output Voltage on the same axis. Click Monitor and adjust the scale on the oscilloscope so there is a complete cycle, so the capacitor fully charges and discharges.

2. Increase the number of points (using the tool on the scope toolbar) to the maximum allowed. Then take a snapshot of both voltages shown. Rename the snapshots "3900pF".

7 Analysis

1. Create a graph with Voltage Ch. A and the Output Voltage vs. time. Select the voltages for the 3900 pF run on the graph.

2. Using the Coordinates Tool, measure the time it takes for the voltage to decay to half of its maximum. This time is the half-life. It may be necessary to reduce the snap-to-pixel distance to 1 in the properties of the Coordinates Tool (right click on the tool to access the properties).

3. Measure the time it takes for the voltage to decay to one-quarter of its maximum. This is two half-lives. Then divide this time by two to find the half-life.

4. Measure the time it takes for the voltage to decay to one-eighth of its maximum. This is three half-lives. Then divide this time by three to find the half-life. Take the average of the three measured values of the half-life. Estimate the precision of the measurement and state it as {half-life ± precision}.

5. Calculate the theoretical half-life given by Equation (12) and compare it to the measured value using a percent difference.

9 Increase the Voltage

Increase the voltage amplitude to 4 V and the offset voltage to 4 V. This makes the square wave positive with an amplitude of 8 V. Keep the circuit components the same. Repeat the procedure and analysis.

10 Decrease the Capacitance

Replace the 3900 pF capacitor with a 560 pF capacitor (capacitor C2 on the UI-5210 board) and repeat the procedure and analysis. The frequency of the signal generator should be changed to 1800 Hz. Return the output voltage to 2 V and the offset voltage to 2 V.

11 Conclusions

1. Summarize how changing the voltage and capacitance changes the half-life.
2. Include the values found for the half-lives and the % differences. Does the theoretical value lie within the range of precision of your measurements? Explain what causes the differences.
3. Did your answers to the Pre-Lab Questions agree with the results?

Set 4V

$T_{\frac{1}{2}} = RC \ln 2$

$= (3900 \times 10^{-12} \frac{C}{V})(47 \times 10^{3} \frac{V}{A}) \ln 2 = 1.27 \times 10^{-4} s = 0.000127$