

Angiao Mai

150/150

Bus190

Midterm Examination 1

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Page 1/2

The exam is open book, notes, and homework, but no computer or smart phone allowed. For full credit, please clearly show your derivations.

1. A decision-maker uses Affordability (A), Quality (Q), and Style (S) as the criteria for selecting a car among three candidates, X, Y, and Z, with the following preferences:

A is moderately to strongly more preferable than Q;

A is extremely more preferable than S;

Q is equally to moderately more preferable than S.

1a. (15 points) Use AHP's 1-9 scale to construct the pair-wise comparison matrix and determine the relative weight (the row average) for each criterion.

	A	Q	S	Row Avg.
X	1	4	9	$\frac{0.735 + 0.727 + 0.715}{3} = 0.737$
Y	$\frac{1}{4}$	1	2	$\frac{0.184 + 0.182 + 0.167}{3} = 0.178$
Z	$\frac{1}{9}$	$\frac{1}{2}$	1	$\frac{0.081 + 0.091 + 0.083}{3} = 0.085$

1b. (35 points) Compute the consistency index of the matrix.

$$(0.737 \times 1) + (0.178 \times 4) + (0.085 \times 9) = 2.214 / 0.737 = 3.004$$

$$(0.737 \times \frac{1}{4}) + (0.178 \times 1) + (0.085 \times 2) = 0.532 / 0.178 = 2.990$$

$$(0.737 \times \frac{1}{9}) + (0.178 \times \frac{1}{2}) + (0.085 \times 1) = 0.256 / 0.085 = 3.012$$

$$(3.004 + 2.990 + 3.012) / 3 = 3.002 = \lambda_{max}$$

$$CI = \frac{3.002 - 3}{2} = 0.001 < 0.05$$

1c. (10 points) Ratings of the three cars with respect to the three criteria are given below:

	A	Q	S	Total Score
0.737 Car X	0.1	0.2	0.6	$0.737 \times 0.1 + 0.178 \times 0.2 + 0.085 \times 0.6 = 0.1603$
0.178 Car Y	0.3	0.6	0.3	$0.737 \times 0.3 + 0.178 \times 0.6 + 0.085 \times 0.3 = 0.3534$
0.085 Car Z	0.6	0.2	0.1	$0.737 \times 0.6 + 0.178 \times 0.2 + 0.085 \times 0.1 = 0.4863$

Use the weights for the three criteria from 1a to compute the total scores of the three cars and summarize them under the Total Score column above.

2. For 4 choices A, B, C, and D, a decision-maker has the following pair-wise comparison matrix and rankings of the confidence in the comparisons:

	A	B	C	D
A	1	3	$\frac{3}{2}$	$\frac{3}{4}$
B	$\frac{1}{3}$	1	$\frac{1}{2}$	$\frac{1}{4}$
C	$\frac{2}{3}$	2	1	$\frac{2}{3}$
D	$\frac{1}{7}$	$\frac{3}{7}$	$\frac{3}{14}$	1

(1) A vs. B Highest confidence

(2) B vs. C

(3) A vs. D

(4) B vs. D

(5) C vs. D

(6) A vs. C Lowest confidence

2a. (30 points) Use the rankings to modify the matrix so that it would become totally consistent.

2b. (5 points) What is the consistency index of the modified matrix?

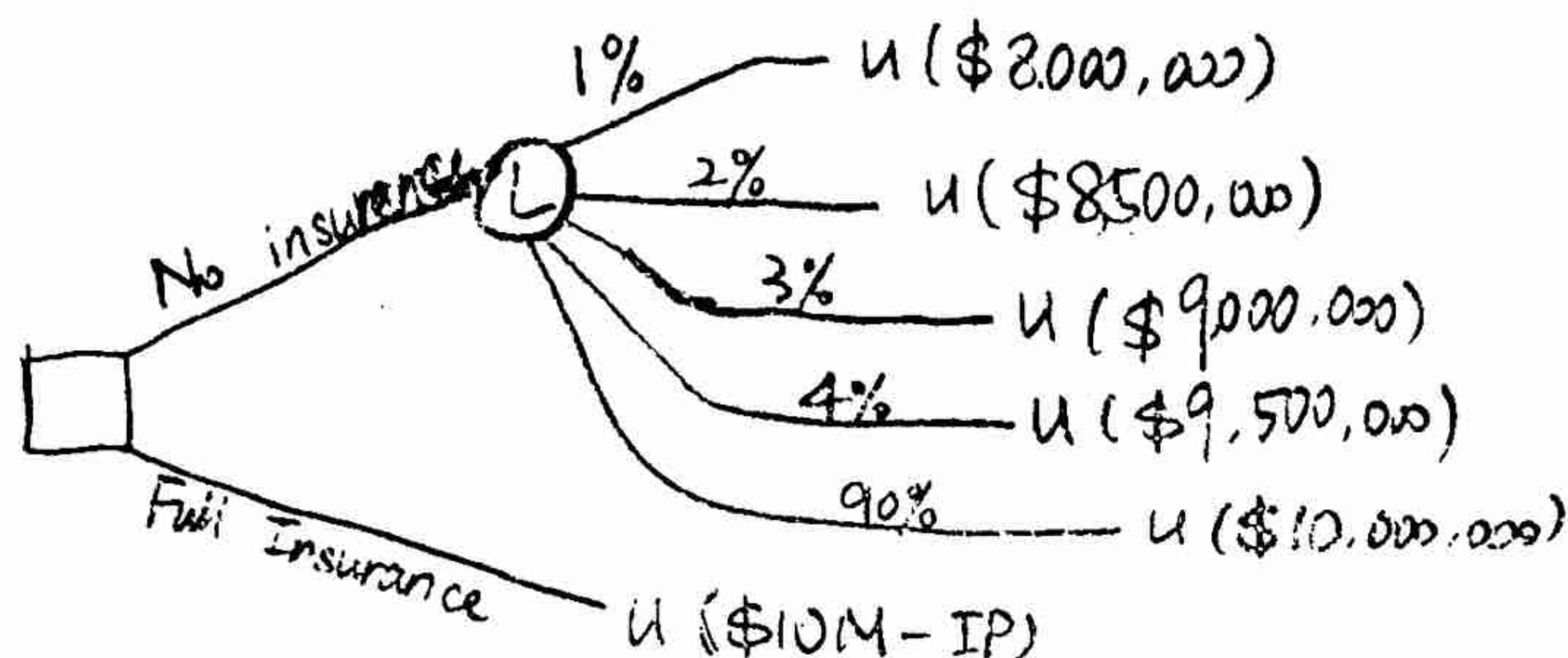
$$CI = 0$$

$$\lambda_{max} = 4$$

3. Through a systematic analysis, John has determined that his utility function for money is equal to the 4th root of the amount of money. John has a total net asset of \$10,000,000, including a house with a structure having a market or replacement value of \$2,000,000. Past annual fire accident statistics indicate that there is a 1% probability that the structure of John's house may be totally destroyed by fire, a 2% probability that it may be 75% destroyed, a 3% probability that it may be half destroyed, a 4% probability that it may be 25% destroyed, and a 90% probability that nothing happens to it.

3a. (15 points) Draw John's decision tree for determining the maximum insurance premium (IP) John would be willing to pay for a \$2,000,000 fire insurance for the structure of his house.

$\sqrt[4]{\$}$
 1% totally
 2% 75% destroyed
 3% 50%
 4% 25%
 90% nothing



3b. (25 points) Use the decision tree to compute the maximum insurance premium that John would be willing to pay for a \$2,000,000 fire insurance for the structure of his house.

$$u(L) = 0.01 \times \sqrt[4]{8,000,000} + 0.02 \times \sqrt[4]{8,500,000} + 0.03 \times \sqrt[4]{9,000,000} + 0.04 \times \sqrt[4]{9,500,000} + 0.9 \times \sqrt[4]{10,000,000}$$

$$= 56.0863$$

$$\sqrt[4]{10M - IP} = 56.0863$$

$$IP = 104,741.08$$

The maximum insurance premium is 104,741.08

3c. (10 points) For a \$2,000,000 fire insurance for the structure of the house, compute the insurance expected payout (IEP).

$$IEP = 0.01 \times 2M + 0.02 \times 1,500,000 + 0.03 \times 1M + 0.04 \times 500,000 + 0.9 \times 0M$$

$$= 100,000$$

3d. (5 points) Compute the risk premium in the insurance premium.

$$\text{Risk premium} = IP - IEP$$

$$= 104,741.08 - 100,000$$

$$= 4741.08$$

1. Amy needs to decide whether to invest in a new product with uncertain profit or a proven product sure to make a net profit of \$5 million. For the new product, she believes that there are 3 possible outcomes: Big Success (BS) for a \$20 million net profit; Small Success (SS) for a \$10 million net profit, or No Success (NS) for a -\$10 million loss. She initially believes that the probabilities for these three outcomes are 0.1, 0.7, and 0.2 respectively. She is considering a new product success forecasting service. After investigating the past history of this service in forecasting products similar to the new product, she found that it has the following track record in terms of conditional probabilities $P(\text{forecast being } +, 0, \text{ or } - \mid \text{actual product being BS, SS, or NS})$:

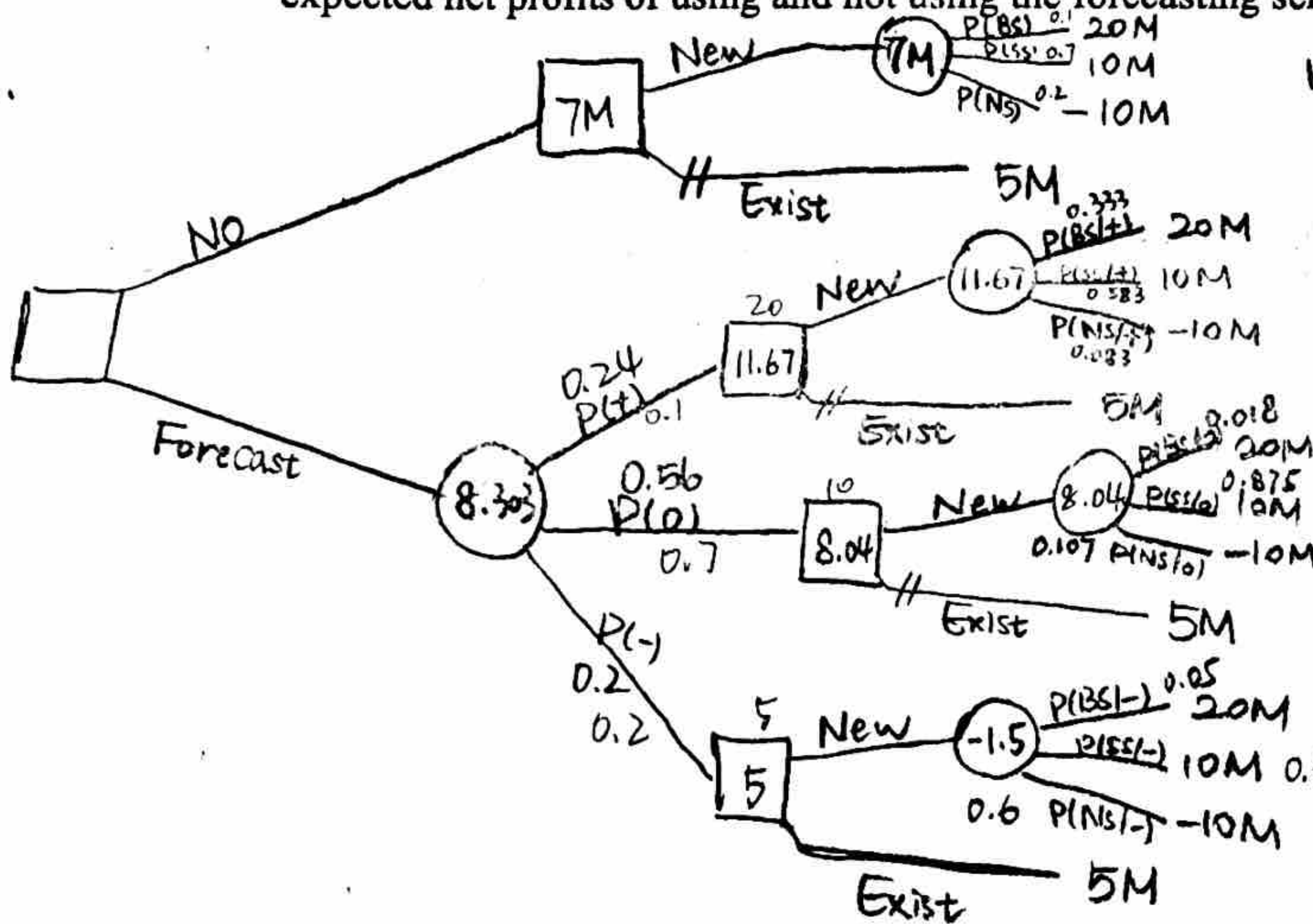
Actual Product Success Being	Forecast Being		
	+	0	-
BS	0.8	0.1	0.1
SS	0.2	0.7	0.1
NS	0.1	0.3	0.6

1a. (15 points) Draw the 2-stage decision tree for whether to hire the forecasting service and which product to invest, with the appropriate labeling of various probabilities and outcome values.

1b. (40 points) Compute all probabilities on the decision tree.

1c. (15 points) Determine the expected value of the forecasting service; i.e., the difference between the expected net profits of using and not using the forecasting service.

1a.



1b.

	Initial Prt	$P(+ EI)$	Joint	$P(EI +)$
BS	0.1	0.1×0.8	0.08	0.08 / 0.24
SS	0.7	0.7×0.2	0.14	0.14 / 0.24
NS	0.2	0.2×0.1	0.02	0.02 / 0.24
			$P(+)=0.24$	

	Initial Prt	$P(0 EI)$	Joint	$P(EI 0)$
BS	0.1	0.1×0.1	0.01	0.01 / 0.56
SS	0.7	0.7×0.7	0.49	0.49 / 0.56
NS	0.2	0.2×0.3	0.06	0.06 / 0.56
			$P(0)=0.56$	

	Initial Prt	$P(- EI)$	Joint	$P(EI -)$
BS	0.1	0.1×0.1	0.01	0.01 / 0.2
SS	0.7	0.7×0.1	0.07	0.07 / 0.2
NS	0.2	0.2×0.6	0.12	0.12 / 0.2
			$P(-)=0.2$	

1c. EV of $I = 8.303 - 7 = 1.303$

1d. (15 points) Determine the expected value of the perfect forecasting service, i.e., one with $P(+|BS)=P(0|SS)=P(-|NS)=1$ and all other probabilities equal to 0. Because these conditional probabilities are very simple, you can easily re-compute all the probabilities on the decision tree

$$0.1 \times 20M + 0.7 \times 10M + 0.2 \times 5M = 10M$$

$$EV \text{ of PI} = 10 - 7 = 3$$

2. The payoff matrix for the Union in a negotiation with the Management for the annual percentage pay raises is given below:

	M1	M2	M3	M4	M5
U1	1	1	1	4	1
U2	1	2	2	3	1
U3	2	3	4	2	2
U4	1	4	4	1	1
U5	2	4	3	2	1

2a. (20 points) Reduce the matrix to the irreducible form and construct all the square submatrices.

2b. (30 points) Use the submatrices to find the best strategy and the value of the game for the player with the greater number of available strategies in the irreducible matrix.

	M3	M4	Q1	Q2	Q3
U1	1	4	P1 [1 4]	P2 [2 3]	P1 [1 4]
U2	2	3	P2 [2 3]	P3 [4 2]	P3 [4 2]
U3	4	2			

$$2P_2 + 4P_3 = 3P_2 + 2P_3$$

$$P_1 + 4P_3 = 4P_1 + 2P_3$$

$$P_2 + P_3 = 1$$

$$P_2 - 2P_3 = 0$$

$$3P_3 = 1$$

$$P_3 = \frac{1}{3}$$

$$P_2 = \frac{2}{3}$$

$$\frac{4}{3} + \frac{4}{3} = 2.67$$

$$2.67 \quad -2.67$$

$$P_1 + P_3 = 1$$

$$3P_1 - 2P_3 = 0$$

$$5P_3 = 3$$

$$P_3 = \frac{3}{5}$$

$$P_1 = \frac{2}{5}$$

$$\frac{2}{5} + \frac{12}{5} = 2.8$$

$$2.8 \quad -2.8$$

Q3 is the best strategy

2c. (15 points) Find the best strategy and the value of the game for the opponent for the sub-game chosen by the player having more strategies.

	(Q1)	(Q2)
P1	1	4
P3	4	2

$$1q_1 + 4q_2 = 4q_1 + 2q_2$$

$$q_1 + q_2 = 1$$

$$5q_2 = 3$$

$$q_2 = \frac{3}{5}$$

$$q_1 = \frac{2}{5}$$

$$\text{Best Strategy} = 1 \times \frac{3}{5} + 4 \times \frac{2}{5} = 2.8, -2.8$$