1) Simplify the following

a. $\frac{\frac{(b-1)!}{(a-2)!}}{\frac{(b+1)!}{(a+1)!}}$

b. $\frac{\Gamma(a+b+1)\Gamma(a)}{\Gamma(a+2)\Gamma(a+b-2)}$

c. $Beta(6,7) \Big/ Beta(10,12)$

d. $Beta(a+2,b) \Big/ Beta(a,b-1)$

e. $\frac{\frac{12!}{16!}}{\frac{14!}{10!}}$

f. $\binom{a-1}{b} \Big/ \binom{b-1}{a}$

g. $\frac{\Gamma(12)\Gamma(8)}{\Gamma(7)\Gamma(13)}$

2) An automobile manufacturer has three factories: A, B, and C. They produce 33%, 43%, and 24% respectively, of a specific model of car. 25% of the cars produced in factory A are white, 45% of those produced in factory B are white, and 25% produced in factory C are white.
   a) If an automobile produced by the company is selected at random, find the probability that it is white.
   b) Given that an automobile selected at random is white, find the probability that it came from factory A.
   c) Given that an automobile selected at random is white, find the probability that it came from factory C.
   d) Given that an automobile selected at random is white, find the probability that it came from factory B.

3) Find the value of K hence the mean and variance of the following discrete random variable X given their respective probabilities

   a) X can take values 0, 2, 4 with the following probabilities $Pr\{X=0\} = K^2$, $Pr\{X=2\} = -0.2K$, $Pr\{X=24\} = 0.01$

b) X can take values 18, 22, 24 with the following probabilities Pr{X=18} =3K, Pr{X=22} = K, Pr{X=24} =2K

c) X can take values 1, 2, 3, 4 with the following probabilities Pr{X=1} =K, Pr{X=2} = 3K, Pr{X=3} =2K, P{X=4}=4K

d) X can take values 1, 2, 4, 6 with the following probabilities Pr{X=1} =K^2, Pr{X=2} = -0.2K, Pr{X=4} =-0.4K, Pr{X=4}=0.09

e) X can take values 0, 1, 2, 3 with the following probabilities Pr{X=0} =K, Pr{X=1} = 0.2K, Pr{X=2} =0.3K, Pr{X=3}=0.1K

4) Derive the mean and variance of

a. Uniform distribution

$$f(x) = \frac{1}{n-m} \qquad m < x < n, \qquad m > 0, n > 0$$

b. Bernoulli distribution

$$\Pr\{X = x\} = P^x(1-p)^{x-1}, x = 0,1$$

c. Poisson distribution

$$\Pr\{X = x\} = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0,1, \ldots$$

d. Gamma distribution

$$f(y) = \frac{6^{\propto} y^{\propto -1} e^{-6\propto}}{\Gamma(\propto)}, \qquad y > 0, \propto > 0$$

e. Geometric distribution

$$\Pr\{X = x\} = p(1-p)^{x-1}, x = 1,2, \ldots$$

5) The random variable y has probability density function f(y) given by

$$f(x) = \begin{cases} ky(4-y) & 0 < y < 3 \\ 0 & Otherwise \end{cases}$$

Find the value of K hence the mean and variance of y

6) Determine the value of θ hence the mean and the variance of the following distribution function of a continuous random variable X.

$$f(x) = \begin{cases} x^3(2x+\theta) & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

7) Consider the following probability density function of a continuous random variable x

$$f(x) = \begin{cases} \frac{1}{5}(K - x) & 0 < x < k \\ 0 & elsewhere \end{cases}$$

Find the value of k, hence the mean and variance of x

8) The monthly demand for gas consumption at South California Gas Station (in thousands of gallons) is given by the following probability density function

$$f(x) = \begin{cases} \beta\left(x - \frac{1}{3}\right)(2 - x) & \frac{1}{2} < x < 2 \\ 0 & elsewhere \end{cases}$$

Find the value of β hence the mean and the variance of x

9) The random variable x has probability density function f(x) given by

$$f(x) = \begin{cases} k(x + 3x^2) & 0 < x < 5 \\ 0 & Otherwise \end{cases}$$

Find the value of K hence the mean and variance of x

10) Use the MGF technique to find the mean and variance of the following distributions

a) Binomial distribution

$$\Pr\{X = x\} = \binom{n}{x} p^x (1 - p)^{n-x}, x = 0,1,2, \dots, n$$

b) Normal distribution

$$f(x) = \frac{1}{5\sqrt{2\pi}} \exp -\frac{1}{2}\left(\frac{x - 15}{5}\right)^2, \quad -\infty < x < \infty$$

c) Gamma distribution

$$f(y) = \frac{y^{\propto - 1} e^{-\propto/8}}{\Gamma(\propto) 8^{\propto}}, \quad y > 0, \propto > 0$$

d) Exponential distribution

$$f(x) = \frac{\mu}{4} \exp\left(-\frac{\mu}{4} x\right), \quad x > 0$$

e) Geometric distribution

$$\Pr\{X = x\} = p(1 - p)^{x-1}, x = 1,2, \dots$$

11) Suppose the random variable X has the following MGF

$$M_x(t) = (0.23e^{2t} + 0.77)^{24}$$

Find the mean and the variance of X

12) Suppose the random variable X has the following MGF

$$M_x(t) = \frac{1}{(1-2t)}\exp(\mu t)$$

Find the mean and the variance of X

13) Suppose the random variable X has the following MGF

$$M_x(t) = (1-3t)^2\exp(-5t)$$

Find the mean and the variance of X

14) If the MGF of variable X is given as $0.5 + 0.5e^{2t}$ and variable Y as $(1+5t)^2$. Let the random variable Z be defined as Z=4X+Y. Find the mean and variance of Z

15) If the MGF of variable X is given as e^{4t} and variable Y as $(1+5t)^2$. Let the random variable Z be defined as Z=X+2Y. Find the mean and variance of Z

16) If the MGF of variable X is given as $(1+2t)e^{4t}$ and variable Y as $(1+5t)^2$. Let the random variable Z be defined as Z=X+5Y. Find the mean and variance of Z

17) Let X and Y be jointly continuous random variables with joint PDF

$$f(x) = \begin{cases} 40e^{-(4x+10y)} & x>0, y>0 \\ 0 & elsewhere \end{cases}$$

a) Are X and Y independent?
b) Find E[Y|X>2].
c) Find P(X>Y).

18) Let X and Y be two jointly continuous random variables with the following joint PDF

$$f(x,y) = \begin{cases} m(5x-y^3) & 0<x<1, \quad 0<y<1, \quad m>0 \\ 0 & Otherwise \end{cases}$$

Find

a. The value of c
b. Marginal distributions f(x) and f(y)
c. The variance covariance matrix

19) Suppose 35% of families own a dog, 45% of families own a cat, and 20% of the families that have a dog also have a cat. A family is chosen at random and found to have a cat. What is the probability they also own a dog?

20) Use the MGF technique to find the mean and variance of Gamma distribution with the following format

$$f(x) = \frac{5^{\propto} x^{\alpha-1} e^{-5x}}{\Gamma(\alpha)} \qquad x > 0, \alpha > 0$$

21) Suppose the random variable X has the following MGF

$$M_x(t) = (0.41e^{4t} + 0.69)^{12}$$

Find the mean and the variance of X

22) If the MGF of variable X is given as $e^{3(e^t-1)}$ and variable Y as $(0.4 + 0.6e^t)^9$. Let the random variable Z be defined as Z=X+Y. Find the mean and variance of Z

23) Let X and Y be jointly continuous random variables with joint PDF

$$f(x) = \begin{cases} 8e^{-(2x+4y)} & x > 0, y > 0 \\ 0 & elsewhere \end{cases}$$

a. Are X and Y independent?
b. Find E[Y|X>2].
c. Find P(Y>X).

24) In New York State, 71% of all teenagers own a skateboard and 32% of all teenagers own a skateboard and roller blades. What is the probability that a teenager owns roller blades given that the teenager owns a skateboard?

25) The probability that Tom will score above 85% on a mathematics test is 0.65. What is the probability that he will score above 85%

i. Exactly 2 of the 14 possible tests this quarter?
ii. At least 2 of the 14 possible tests this quarter?
iii. At most 3 of the 14 possible tests this quarter?