1. A country ("Home") is populated with workers who produce either food (F) or clothing (C). There are 200 workers producing food and 100 producing clothing; these numbers are fixed (no adjustment in this short-run model). Each food worker produces 6 units of food and each clothing worker produces 3 units of clothing. Workers own the output they produce and can trade with another. All workers share the same preferences over food and clothing represented by the utility function: $U(D_C, D_F) = D_C^{1/3} D_F^{2/3}$

   (a) What is the autarky price (the relative price of food and clothing) in this economy? Hint: recall that agents with homothetic Cobb-Douglas utility functions spend fixed shares of their income on each good, corresponding to the exponent on the good.

   (b) In autarky, how many units of food and clothing will be consumed by a clothing worker? By a food worker?

   (c) There also exists another country ("Foreign") that looks surprisingly like the Home country except that there are 600 food workers and 300 clothing workers in this country. These workers have different (lower) productivity levels: a food worker can produce only 1 unit of food while a clothing worker can only produce 2 units of clothing. The workers in Foreign share the same preferences as those in Home. Answer questions (a) and (b) for the Foreign economy in autarky.

   (d) Compare the consumption levels of workers in both countries. What explains the differences between countries?

   (e) Now assume that these 2 countries open to trade with one another.

      i. What will be the free trade relative price of food and clothing?

      ii. Describe the pattern of trade (who exports what, and how much). Verify that the export supply matches the import demand for both goods.

      iii. Calculate the new consumption levels of both types of workers in both countries. For each type of worker in both countries, asses whether they are better off in the new trade equilibrium. Could you have predicted this pattern before calculating the new consumption levels and the free trade price equilibrium? How?

      iv. Now assume that all workers belong to representative "families," each containing 2 food workers and 1 clothing worker. Describe how the consumption patterns (the number of units of food and clothing consumed) of the "families" change between autarky and the trade equilibrium. Are the "families" in both countries unambiguously better off? Why should you not be surprised by this result?

   (f) *Without* redoing the calculations in (e), describe how your answers would change if the productivity of both types of Foreign workers were multiplied by 10.

(g) Return to the trade equilibrium described in (e). If workers can switch occupations in the long run, what will happen to the number of workers in each industry (only describe the direction, not the magnitude, of the change)?

2. In this exercise, you will show that $\frac{X_1+X_2}{Y_1+Y_2}$ is a weighted average of $\frac{X_1}{Y_1}$ and $\frac{X_2}{Y_2}$ as stated in class (assuming all variables are positive). A weighted average of two positive numbers X and Y is given by $aX + (1-a)Y$ for some fraction a between 0 and 1 (if a= 1/2, this is just a plain average).

   (a) Show that a weighted average of two numbers X and Y must lie between X and Y. (Hint: one number has to be larger than the other, so let $X < Y$ without loss of generality. If $X < Y$, what does this imply about $aX + (1-a)Y$ relative to X? To Y?)

   (b) Show that $\frac{X_1+X_2}{Y_1+Y_2}$ can be written as a weighted average of $\frac{X_1}{Y_1}$ and $\frac{X_2}{Y_2}$. (Hint: you can break this into a sum of two separate fractions. Multiplying both the denominator and numerator of a fraction leaves the total unchanged—how might you use this to rewrite in terms of $\frac{X_1}{Y_1}$ and $\frac{X_2}{Y_2}$?)

3. Two agricultural economies, "Home" and "Foreign," produce corn (C) and wheat (W). Home has an endowment of 200 fields, which can produce either 2 units of corn or 2 units of wheat. Foreign has an endowment of 300 fields, which can produce either 1 unit of corn or 2 units of wheat. Consumers in both economies have identical homothetic preferences represented by the utility function: $U(D_C, D_W) = D_C^{1/2} D_W^{1/2}$

   (a) Draw the production possibility frontiers for both economies (draw the production of corn on the X-axis).

   (b) Let $p = p_C/p_W$ denote the relative price of corn. On a graph with p on the Y-axis and relative quantities C/W on the X-axis, draw the *relative* demand and supply curves for both economies in autarky (use a different diagram for each economy). Make sure to indicate the coordinates of the point where both curves intersect. Do these curves depend on the economies' endowments (the number of fields they have)?

   (c) Now assume that these economies open up to international trade. Draw the *world* relative demand and supply curves on a separate diagram. Indicate on the diagram the coordinates of the intersection point between the two curves and the level of the autarky relative prices. (Hint 1: Do you need to solve for the world relative demand curve again? Hint 2: How do countries specialize at different price levels? Does the price specialization "jump" when different countries specialize?)

   (d) What is the pattern (indicate the quantity levels) of production and consumption in both countries in this trade equilibrium?

   (e) Now assume that Home's productivity changes: each field can produce the same amount of corn and wheat, but this amount is now a variable $a > 0$ (previously a = 2).

      i. Solve for the world equilibrium price again as in (c), but now letting the answers depend on a. How does p change with a?

      ii. How do Home's terms of trade change as it experiences productivity growth? Explain, intuitively, why becoming a larger share of the world economy affects the world trade price, and thus comparative advantage. (Hint: Recall that world relative supply can be written as a weighted sum of each country's relative supply. Is domestic relative supply changing in either country? Are the weights changing, according to the answer to problem 2?)

      iii. How does aggregate welfare in Home change with a?

4. Consider a general model of Ricardian trade with 2 countries (Home and Foreign) and 2 goods (Clothing and Food): unit labor costs are a_C and a_F in Home and a_C^* and a_F^* in Foreign. Home and Foreign are endowed, respectively, with L and L^* units of labor. Workers in both countries have the same preferences represented by a general Cobb-Douglas utility function $U(D_C; D_F) = D_C^{\beta_C} D_F^{\beta_F}$ where β_C and β_F are positive constants. Further assume that Home has a comparative advantage in Clothing: $\frac{a_C}{a_F} < \frac{a_C^*}{a_F^*}$.

(a) Let $p^T = \frac{p_C^T}{p_F^T}$ be the relative price of clothing under free trade. What will be the range of possible prices for p^T that will lead to complete specialization in both countries? In this scenario, what will the world production quantities be? For the remainder of this question, assume that this relationship holds and that both countries are completely specialized in the free trade equilibrium.

(b) Write p^T in terms of the parameters of the model (the a's, the $\beta's$, and the L's). (Hint: In equilibrium, world demand quantities must equal supply. We determined what world supply levels have to be in the last question).

(c) How would a 1% increase in Home's labor force L affect p^T? How would a 1% increase in Home's productivities (a_C and a_F both increase by 1%) affect p^T?

(d) Write the relative wage $\frac{w}{w^*}$ as a function of the parameters of the model. How would a 1% increase in Home's labor force L affect $\frac{w}{w^*}$? How would a 1% increase in Home's productivities (a_C and a_F both *decrease* by 1%) affect $\frac{w}{w^*}$?

(e) Describe how an increase in Home's labor force L or an increase its productivity will affect welfare in both countries. In particular, how does purchasing power for each good (wage divided by world trade price) change in each case?