Graphical Solution of Linear Equations in One Variable Suppose that y_1 and y_2 are linear expressions in x. We can solve the equation $y_1 = y_2$ graphically as follows (assuming it has a unique solution).

1. Rewrite the equation as $y_1 - y_2 = 0$.
2. Graph the linear function $y_3 = y_1 - y_2$.
3. Find the x-intercept of the graph of the function y_3. This x-value is the solution of $y_1 = y_2$.

Some calculators use the term *zero* to identify the x-value of an x-intercept. In general, if $f(a) = 0$, then a is a **zero** of f.

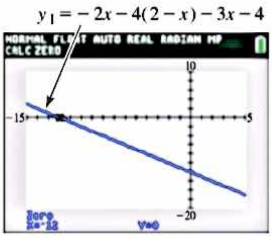


Figure 51

EXAMPLE 8 Solving an Equation with a Graphing Calculator

Use a graphing calculator to solve $-2x - 4(2 - x) = 3x + 4$.

SOLUTION We write an equivalent equation with 0 on one side.

$$-2x - 4(2 - x) - 3x - 4 = 0 \quad \text{Subtract } 3x \text{ and } 4.$$

Then we graph $y = -2x - 4(2 - x) - 3x - 4$ to find the x-intercept. The standard viewing window cannot be used because the x-intercept does not lie in the interval $[-10, 10]$. As seen in **Figure 51,** the solution of the equation is -12, and the solution set is $\{-12\}$.

Now Try Exercise 69.

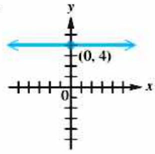
2.5 Exercises

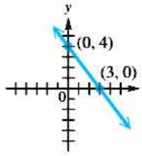
CONCEPT PREVIEW *Fill in the blank(s) to correctly complete each sentence.*

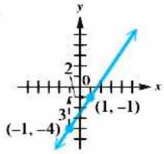
1. The graph of the line $y - 3 = 4(x - 8)$ has slope ____ and passes through the point $(8, \_\_)$.
2. The graph of the line $y = -2x + 7$ has slope ____ and y-intercept ____.
3. The vertical line through the point $(-4, 8)$ has equation ____ $= -4$.
4. The horizontal line through the point $(-4, 8)$ has equation ____ $= 8$.
5. Any line parallel to the graph of $6x + 7y = 9$ must have slope ____.
6. Any line perpendicular to the graph of $6x + 7y = 9$ must have slope ____.

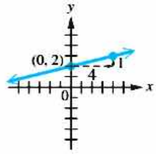
CONCEPT PREVIEW *Match each equation with its graph in A–D.*

7. $y = \frac{1}{4}x + 2$
8. $4x + 3y = 12$
9. $y - (-1) = \frac{3}{2}(x - 1)$
10. $y = 4$

A.


B.


C.


D.


Write an equation for each line described. Give answers in standard form for Exercises 11–20 and in slope-intercept form (if possible) for Exercises 21–32. ***See Examples 1–4.***

11. through $(1, 3)$, $m = -2$

12. through $(2, 4)$, $m = -1$

13. through $(-5, 4)$, $m = -\frac{3}{2}$

14. through $(-4, 3)$, $m = \frac{3}{4}$

15. through $(-8, 4)$, undefined slope

16. through $(5, 1)$, undefined slope

17. through $(5, -8)$, $m = 0$

18. through $(-3, 12)$, $m = 0$

19. through $(-1, 3)$ and $(3, 4)$

20. through $(2, 3)$ and $(-1, 2)$

21. x-intercept $(3, 0)$, y-intercept $(0, -2)$

22. x-intercept $(-4, 0)$, y-intercept $(0, 3)$

23. vertical, through $(-6, 4)$

24. vertical, through $(2, 7)$

25. horizontal, through $(-7, 4)$

26. horizontal, through $(-8, -2)$

27. $m = 5$, $b = 15$

28. $m = -2$, $b = 12$

29. through $(-2, 5)$ having slope -4

30. through $(4, -7)$ having slope -2

31. slope 0, y-intercept $\left(0, \frac{3}{2}\right)$

32. slope 0, y-intercept $\left(0, -\frac{5}{4}\right)$

33. ***Concept Check*** Fill in each blank with the appropriate response:

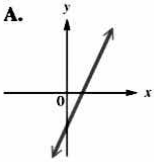
The line $x + 2 = 0$ has x-intercept ________. It ________ (does/does not) have a y-intercept. The slope of this line is ________ (0/undefined).

The line $4y = 2$ has y-intercept ________. It ________ (does/does not) have an x-intercept. The slope of this line is ________ (0/undefined).

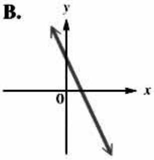
34. ***Concept Check*** Match each equation with the line that would most closely resemble its graph. (*Hint:* Consider the signs of m and b in the slope-intercept form.)

(a) $y = 3x + 2$ **(b)** $y = -3x + 2$ **(c)** $y = 3x - 2$ **(d)** $y = -3x - 2$

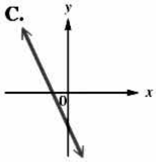
A.



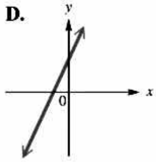
B.



C.



D.



Find the slope and y-intercept of each line, and graph it. ***See Example 3.***

35. $y = 3x - 1$

36. $y = -2x + 7$

37. $4x - y = 7$

38. $2x + 3y = 16$

39. $4y = -3x$

40. $2y = x$

41. $x + 2y = -4$

42. $x + 3y = -9$

43. $y - \frac{3}{2}x - 1 = 0$

44. ***Concept Check*** The table represents a linear function f.

(a) Find the slope of the graph of $y = f(x)$.

(b) Find the y-intercept of the line.

(c) Write an equation for this line in slope-intercept form.

x	y
-2	-11
-1	-8
0	-5
1	-2
2	1
3	4

2.5 Equations of Lines and Linear Models

- Point-Slope Form
- Slope-Intercept Form
- Vertical and Horizontal Lines
- Parallel and Perpendicular Lines
- Modeling Data
- Graphical Solution of Linear Equations in One Variable

Point-Slope Form

The graph of a linear function is a straight line. We now develop various forms for the equation of a line.

Figure 43 shows the line passing through the fixed point (x_1, y_1) having slope m. (Assuming that the line has a slope guarantees that it is not vertical.) Let (x, y) be any other point on the line. Because the line is not vertical, $x - x_1 \neq 0$. Now use the definition of slope.

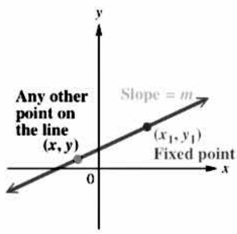


Figure 43

$$m = \frac{y - y_1}{x - x_1} \qquad \text{Slope formula}$$

$$m(x - x_1) = y - y_1 \qquad \text{Multiply each side by } x - x_1.$$

$$y - y_1 = m(x - x_1) \qquad \text{Interchange sides.}$$

This result is the *point-slope form* of the equation of a line.

LOOKING AHEAD TO CALCULUS

A standard problem in calculus is to find the equation of the line tangent to a curve at a given point. The derivative (see *Looking Ahead to Calculus* earlier in this chapter) is used to find the slope of the desired line, and then the slope and the given point are used in the point-slope form to solve the problem.

Point-Slope Form

The **point-slope form** of the equation of the line with slope m passing through the point (x_1, y_1) is given as follows.

$$\mathbf{y - y_1 = m(x - x_1)}$$

EXAMPLE 1 Using the Point-Slope Form (Given a Point and the Slope)

Write an equation of the line through the point $(-4, 1)$ having slope -3.

SOLUTION Here $x_1 = -4$, $y_1 = 1$, and $m = -3$.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - 1 = -3[x - (-4)] \qquad x_1 = -4,\ y_1 = 1,\ m = -3$$

$$y - 1 = -3(x + 4) \qquad \text{Be careful with signs.}$$

$$y - 1 = -3x - 12 \qquad \text{Distributive property}$$

$$y = -3x - 11 \qquad \text{Add 1.}$$

✔ **Now Try Exercise 29.**

EXAMPLE 2 Using the Point-Slope Form (Given Two Points)

Write an equation of the line through the points $(-3, 2)$ and $(2, -4)$. Write the result in standard form $Ax + By = C$.

SOLUTION Find the slope first.

$$m = \frac{-4 - 2}{2 - (-3)} = -\frac{6}{5} \qquad \text{Definition of slope}$$

The slope m is $-\frac{6}{5}$. Either the point $(-3, 2)$ or the point $(2, -4)$ can be used for (x_1, y_1). We choose $(-3, 2)$.

$$
\begin{aligned}
y - y_1 &= m(x - x_1) && \text{Point-slope form} \\
y - 2 &= -\frac{6}{5}[x - (-3)] && x_1 = -3,\ y_1 = 2,\ m = -\tfrac{6}{5} \\
5(y - 2) &= -6(x + 3) && \text{Multiply by 5.} \\
5y - 10 &= -6x - 18 && \text{Distributive property} \\
6x + 5y &= -8 && \text{Standard form}
\end{aligned}
$$

Verify that we obtain the same equation if we use $(2, -4)$ instead of $(-3, 2)$ in the point-slope form.

✔ **Now Try Exercise 19.**

NOTE The lines in **Examples 1 and 2** both have negative slopes. Keep in mind that a slope of the form $-\frac{A}{B}$ may be interpreted as either $\frac{-A}{B}$ or $\frac{A}{-B}$.

Slope-Intercept Form As a special case of the point-slope form of the equation of a line, suppose that a line has y-intercept $(0, b)$. If the line has slope m, then using the point-slope form with $x_1 = 0$ and $y_1 = b$ gives the following.

$$
\begin{aligned}
y - y_1 &= m(x - x_1) && \text{Point-slope form} \\
y - b &= m(x - 0) && x_1 = 0,\ y_1 = b \\
y - b &= mx && \text{Distributive property} \\
y &= \underbrace{m}_{\text{Slope}}x + \underbrace{b}_{\text{The } y\text{-intercept is } (0,\, b).} && \text{Solve for } y.
\end{aligned}
$$

Because this result shows the slope of the line and indicates the y-intercept, it is known as the *slope-intercept form* of the equation of the line.

Slope-Intercept Form

The **slope-intercept form** of the equation of the line with slope m and y-intercept $(0, b)$ is given as follows.

$$y = mx + b$$

EXAMPLE 3 Finding Slope and y-Intercept from an Equation of a Line

Find the slope and y-intercept of the line with equation $4x + 5y = -10$.

SOLUTION Write the equation in slope-intercept form.

$$
\begin{aligned}
4x + 5y &= -10 \\
5y &= -4x - 10 && \text{Subtract } 4x. \\
y &= \underbrace{-\frac{4}{5}}_{m}x \underbrace{- 2}_{b} && \text{Divide by 5.}
\end{aligned}
$$

The slope is $-\frac{4}{5}$, and the y-intercept is $(0, -2)$. ✔ **Now Try Exercise 37.**

NOTE Generalizing from **Example 3,** we see that the slope m of the graph of the equation

$$Ax + By = C$$

is $-\frac{A}{B}$, and the y-intercept is $\left(0, \frac{C}{B}\right)$.

EXAMPLE 4 Using the Slope-Intercept Form (Given Two Points)

Write an equation of the line through the points $(1, 1)$ and $(2, 4)$. Then graph the line using the slope-intercept form.

SOLUTION In **Example 2,** we used the *point-slope form* in a similar problem. Here we show an alternative method using the *slope-intercept form*. First, find the slope.

$$m = \frac{4-1}{2-1} = \frac{3}{1} = 3 \quad \text{Definition of slope}$$

Now substitute 3 for m in $y = mx + b$ and choose one of the given points, say $(1, 1)$, to find the value of b.

$$y = mx + b \quad \text{Slope-intercept form}$$
$$1 = 3(1) + b \quad m = 3, x = 1, y = 1$$
$$\text{The } y\text{-intercept is } (0, b). \rightarrow b = -2 \quad \text{Solve for } b.$$

The slope-intercept form is

$$y = 3x - 2.$$

The graph is shown in **Figure 44.** We can plot $(0, -2)$ and then use the definition of slope to arrive at $(1, 1)$. Verify that $(2, 4)$ also lies on the line.

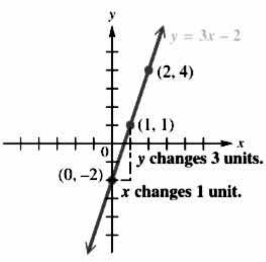


Figure 44

✔ **Now Try Exercise 19.**

EXAMPLE 5 Finding an Equation from a Graph

Use the graph of the linear function f shown in **Figure 45** to complete the following.

(a) Identify the slope, y-intercept, and x-intercept.

(b) Write an equation that defines f.

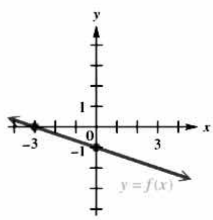


Figure 45

SOLUTION

(a) The line falls 1 unit each time the x-value increases 3 units. Therefore, the slope is $\frac{-1}{3} = -\frac{1}{3}$. The graph intersects the y-axis at the y-intercept $(0, -1)$ and the x-axis at the x-intercept $(-3, 0)$.

(b) The slope is $m = -\frac{1}{3}$, and the y-intercept is $(0, -1)$.

$$y = f(x) = mx + b \quad \text{Slope-intercept form}$$
$$f(x) = -\frac{1}{3}x - 1 \quad m = -\frac{1}{3}, b = -1$$

✔ **Now Try Exercise 45.**