1. For a fully discrete whole life insurance of 1000 on (x):
(i) Death is the only decrement.
(ii) The annual net premium is 80.
(iii) Expenses in year 1, payable at the start of the year, are 40% of contract premiums.
(iv) The starting policy reserve is 0 and the policy reserve at the end of the first year is 16.84.
(v) $i = 0.10$ (vi) $1000\,{}_1V_x = 40$
Calculate the level annual contract premium.
A) 100 B) 102 C) 104 D) 106 E) 108

2. For a special fully discrete 2-year term insurance on (x):
(i) The pure endowment is 2000.
(ii) The death benefit for year k is $(1000k)$ plus the net reserve at the end of year k, $k = 1, 2$.
(iii) π is the level annual net premium.
(iv) $i = 0.08$
(v) $p_{x+k-1} = 0.9\,,\ k = 1, 2$
Calculate π
A) 137 B) 157 C) 177 D) 197 E) 207

3. For a group of N individuals age x, you are given:
(i) The future lifetimes are independent.
(ii) Each individual is paid 500 at the beginning of each year, if living.
(iii) $A_x = 0.369131$
(iv) ${}^2A_x = 0.1774113$
(v) $i = 0.06$
Using the normal approximation, an aggregate fund of 1,429,440 is needed at inception in order to be 90% certain of having enough money to pay the life annuities. Find N.
A) 200 B) 225 C) 250 D) 275 E) 300

4. For a 10-payment, 20-year term insurance of 100,000 on (70):
(i) Death benefits are payable at the moment of death.
(ii) Contract premiums of 1600 are payable annually at the beginning of each year for 10 years.
(iii) $i = 0.05$
(iv) L is the loss random variable at the time of issue.
(v) Mortality follows the LTAM Life Table with the assumption of uniform distribution of deaths in each year of age.
Calculate the median value of L.
A) Less than 24,000 B) At least 24,000 but less than 25,000
C) At least 25,000 but less than 26,000 D) At least 26,000 but less than 27,000
E) At least 26,000

9. You are given:

(i) the following select-and-ultimate table with a 3-year select period:

x	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
60	.09	.11	.13	.15	63
61	.10	.12	.14	.16	64
62	.11	.13	.15	.17	65
63	.12	.14	.16	.18	66
64	.13	.15	.17	.19	67

(ii) $i = .03$

Calculate ${}_{2|}A_{[60]:\overline{2}|}$, the actuarial present value of a 2-year deferred 2-year endowment insurance on [60].

A) .66 B) .68 C) .70 D) .72 E) .74

10. For an annuity payable semiannually, you are given:

(i) Deaths are uniformly distributed over each year of age.

(ii) $i^{(2)}/d^{(2)} = 1.035374$.

(iii) $\ddot{a}_x^{(2)} = 10.0$

Calculate $\overline{A}_x$.

A) Less than .30 B) At least .30 but less than .31 C) At least .31 but less than .32

D) At least .32 but less than .33 E) At least .33

11. You are given:

- A whole life policy on a life aged 30
- The policy pays 1,000 at the end of the year of death
- Mortality follows the LTAM Life Table
- $i = 5\%$
- Premiums satisfy the equivalence principle with the following payment schedule:

$$P = \begin{cases} x & t = 0, 1, 2, 3, 4 \\ 2x & t = 5, 6, 7, \ldots \end{cases}$$

Calculate x.

A) Less than 1.4 B) At least 1.4, but less than 1.6 C) At least 1.6, but less than 1.8

D) At least 1.8, but less than 2.0 E) At least 2.0

12. For a fully discrete whole life insurance of 1000 on (60), the annual net premium was calculated using the following:

(i) $i = 0.06$

(ii) $1000A_{60} = 369.33$

A particular insured is expected to experience a first- year mortality rate ten times the rate used to calculate the annual net premium. The expected mortality rates for all other years are the ones originally used. The expected loss at issue for this insured, based on the original net premium is 114.3. Calculate q_{60}.

A) .00822 B) .00950 C) .01128 D) .01376 E) .01546

13. You are given the following mortality probabilities for ages x to $x+3$ for calendar year C.
$q_x = .05$, $q_{x+1} = .08$, $q_{x+2} = .12$, $q_{x+3} = .2$
R is the value of ${}_4p_x$ without mortality improvement factors and S is the value of ${}_4p_x$ with mortality improvement factors, assuming a constant mortality improvement factor $\phi_x = .1$ for all x. Calculate $\frac{R}{S}$.
A) Less than .800 B) At least .800 but less than .825 C) At least .825 but less than .850
D) At least .850 but less than .875 E) At least .875

15. A mortality model has constant mortality probability $q_y = q$ for all y. The annual effective rate of interest is i. A 10-year fully discrete term insurance with face amount 1 is issued to (x).
Find $\sum_{k=1}^{10} {}_kV$ for this policy in terms of q and i.
A) 0 B) $10q$ C) $a_{\overline{10}|i}$ D) $a_{x:\overline{10}|}$ E) $10vq$

18. A 40 year endowment insurance policy having face amount $200,000 is issued to (35). The policy has first year expenses of $200 plus 25% of the expense-loaded premium, and renewal expenses of $50 plus 10% of the expense-loaded premium. There is a settlement expense of $200 in case of death, but if the endowment is paid, there is no settlement expense. You are given (i) $A_{35:\overline{40}|} = .124549$, (ii) $A_{\substack{\;\;1\\35:\overline{40}|}} = .076578$ and (iii) $i = .06$.
If $q_{35} = .001223$ and $q_{36} = .0012735$, what is the second year expense reserve?
A) -420 B) -424 C) -428 D) -432 E) -436

19. (35) buys a special 30-year fully discrete endowment insurance policy with endowment amount 500,000 and with level net premiums payable for 20 years. The death benefit is the terminal reserve for the end of the year in which death occurs for the first 29 years of the policy, and there is no death benefit in the 30th year of the policy. You are given that $q_{64} = .02$ and $i = .04$. Calculate the annual net premium.
A) Less than 10,000 B) At least 10,000 but less than 10,200
C) At least 10,200 but less than 10,400 D) At least 10,400 but less than 10,600
E) At least 10,600

Solutions

1. The accumulation of policy reserve in the 1st year is
$[0+G(.6)](1.1)-1000q_x = p_x \cdot (16.84)$. If we knew the value of q_x, we could find G.
Using the recursive relationship for net reserve for the 1st year, we have
$[0+80](1.1)-1000q_x = p_x(40)$, and solving for q_x results in $q_x = .05$.
Then $.6G(1.1)-1000(.05) = (.95)\cdot(16.84) \rightarrow G = 100$. Answer: A

2. We use the recursive reserve relationship

$$({}_{k-1}V+\pi)(1+i)-(b_k-{}_kV)\cdot q_{x+k-1} = {}_kV .$$

At time 0, ${}_0V=0$, and at time 2 when the policy ends, ${}_2V=0$ since this is a term insurance..
For the first year ($k=1$) , since $b_1 = 1000+{}_1V$, we have
$\pi(1.08)-1000(.1) = {}_1V$, so that ${}_1V = 1.08\pi - 100$.
Then, for the second year , $b_2 = 2000+{}_2V$, so that
$(1.08\pi-100+\pi)(1.08)-2000(.1)=0$.
Solving for π results in $\pi = 137.1$.
An alternative approach uses the following general relationship for reserves:

$${}_mV = \pi\cdot\ddot{s}_{\overline{m}|} - \sum_{k=0}^{m-1} q_{x+k}\cdot(1+i)^{m-k-1}\cdot(b_{k+1}-{}_{k+1}V) \quad .$$

With $m=2$ for this policy, we have $b_1 = 1000+{}_1V$ and $b_2 = 2000+{}_2V$, so that
$0 = \pi\cdot\ddot{s}_{\overline{2}|} - [q_x(1+i)(1000)+q_{x+1}(2000)]$
$= \pi(2.2464)-[(.1)(1.08)(1000)+(.1)(2000)]$ so that $\pi = 137.1$. Answer: A

3. Let S denote the aggregate present value random variable of the N annuities.
Then $S=\sum_{j=1}^{N} W_j$, where each $W_j = 500Y$, and Y is the PVRV for a life annuity-due of 1 per year. Then $E[Y] = \ddot{a}_x = \frac{1-A_x}{d} = \frac{1-.369131}{.06/1.06} = 11.1454$, and
$Var[Y] = \frac{1}{d^2}\cdot[{}^2A_x-(A_x)^2] = 12.8445$.
Then, $E[W]=500E[Y]=5572.7$, $Var[W]=500^2\cdot Var[Y] = 3,211,125$, and
$E[S] = N\cdot E[W] = 5572.7N$, and from the independence of the lives,
$Var[S] = N\cdot Var[W] = 3,211,125N$.

We want the initial fund amount F so that $P[S\le F]=.90$. Applying the normal approximation, we get $P[\frac{S-5572.7N}{\sqrt{3,211,125N}} \le \frac{1,429,400-5572.7N}{\sqrt{3,211,125N}}] = \Phi(\frac{1,439,400-5572.7N}{\sqrt{3,211,125N}}) = .9$.
From the normal table, we get $\frac{1,429,400-5572.7N}{\sqrt{3,211,125N}} = 1.28$.
We can solve a quadratic equation for N, or try each possible answer to find $N=250$.
Answer: C

4. L = PVRV benefit − PVRV premium

The later that the death occurs, the smaller the PV of benefit and the larger the PV of premium received. The median value of L, say L_m would satisfy $P[L \le L_m] = .5$. If we find the point t_m for which ${}_{t_m}p_{70} = .5$, and then find the issue-date loss L_{t_m} based on death at exact time t, then $L \le L_{t_m}$ is equivalent to $t \ge t_m$, which has probability .5.

From the table, $\ell_{70} = 91,082.4$. In order to have ${}_{t_m}p_{70} = .5$, we must have $\frac{\ell_{t_m}}{91,082.4} = .5$, so that $\ell_{t_m} = 45,541.2$. From the table we see that $\ell_{89} = 45,995.6$ and $\ell_{90} = 41,841.1$. Using linear interpolation, we see that under UDD we have $\ell_{89.109} = 45,541.2$. The issue date loss if death occurs at age 89.109 (19.109 years after issue) is $100,000v^{19.109} - 1600\ddot{a}_{\overline{10}|.05} = 26,391$. This is the median of L. Answer: D

9. ${}_{2|}A_{[60]:\overline{2}|} = v^3 \cdot {}_{2|}q_{[60]} + v^4 \cdot ({}_{3|}q_{[60]} + {}_4p_{[60]})$.

${}_{2|}q_{[60]} = {}_2p_{[60]} \cdot q_{[60]+2} = p_{[60]} \cdot p_{[60]+1} \cdot q_{[60]+2} = (1-.09)(1-.11)(.13) = .1053$,

${}_{3|}q_{[60]} = {}_3p_{[60]} \cdot q_{[60]+3} = p_{[60]} \cdot p_{[60]+1} \cdot p_{[60]+2} \cdot q_{63} = (1-.09)(1-.11)(1-.13)(.15) = .1057$

${}_4p_{[60]} = p_{[60]} \cdot p_{[60]+1} \cdot p_{[60]+2} \cdot p_{63} = (1-.09)(1-.11)(1-.13)(.85) = .5989.$

${}_{2|}A_{[60]:\overline{2}|} = .7224$. Answer: D

10. Under the UDD assumption, $\ddot{a}_x^{(m)} = \alpha(m) \cdot \ddot{a}_x - \beta(m)$.

$i^{(2)}/d^{(2)} = 1 + \frac{i^{(2)}}{2} = 1.035374 \to i^{(2)} = .070748$, and $d^{(2)} = .068331$.

Then $i = (1 + \frac{i^{(2)}}{2})^2 - 1 = .0720$ and $d = \frac{i}{1+i} = .067164$

Then, $\alpha(2) = \frac{id}{i^{(2)}d^{(2)}} = 1.000316$ and $\beta(2) = \frac{\frac{i}{i^{(2)}} - 1}{d^{(2)}} = .258984$.

Then, $\ddot{a}_x = \frac{\ddot{a}_x^{(2)} + \beta(2)}{\alpha(2)} = 10.2557$, and $A_x = 1 - d\ddot{a}_x = .3112$.

Finally, under UDD, $\overline{A}_x = \frac{i}{\delta} \cdot A_x = \frac{.0720}{ln(1.072)} \cdot .3112 = .322$. Answer: D

11. From the LTAM Table we have $1000A_{30} = 76.98$.

The APV of the premiums is $2x \cdot \ddot{a}_{30} - x \cdot \ddot{a}_{30:\overline{5}|}$.

From the LTAM Table we have $\ddot{a}_{30} = 19.3834$, and

$\ddot{a}_{30:\overline{5}|} = \ddot{a}_{30} - v^5\,{}_5p_{30}\,\ddot{a}_{35} = \ddot{a}_{30} - {}_5E_{30}\,\ddot{a}_{35}$

$= 19.383 - (.78219)(18.7928) = 4.6839$.

The APV of premiums is $2x(19.3834) - x \cdot (4.6839) = 34.0829x$.

Then $34.0829x = 76.988$, so that $x = 2.26$. Answer: E

12. The original net premium is based on the original insurance value.
$1000P_{60} = \frac{1000dA_{60}}{1-A_{60}} = \frac{1000(.0566)(.36933)}{1-.36933} = 33.145$.
The expected loss at issue is
APV benefit $-$ APV premium $= 1000A'_{60} - 33.145(\frac{1-A'_{60}}{d})' = 114.3$.
It follows that $\quad A'_{60} = .44140$.
Therefore $\quad A_{60} = .36933 = vq_{60} + v(1-q_{60})A_{61}$
and $\quad A'_{60} = .44140 = 10vq_{60} + v(1-10q_{60})A_{61}$
Since there is no change in mortality from age 61, we get
$A_{61} = \frac{.36933 - vq_{60}}{v(1-q_{60})} = \frac{.4414 - 10vq_{60}}{v(1-10q_{60})}$.
This results in a quadratic equation for q_{60}, which we can solve. Alternatively, we can "guess and check" to find that $q_{60} = .01376$. Answer: D

13. $R = .95 \times .92 \times .88 \times .8 = .61529$
(ii) $q(x,C) = .05$, $q(x+1, C+1) = q(x+1, C) \times (1-\phi_{x+1}) = .08 \times .9 = .072$,
$q(x+2, C+2) = q(x+2, C) \times (1-\phi_{x+1})^2 = .12 \times .9^2 = .0972$,
$q(x+3, C+4) = q(x+3, C) \times (1-\phi_{x+1})^3 = .2 \times .9^3 = .1458$,
$S = (1-.05)(1-.072)(1-.0972)(1-.1458) = .67987.$
$\frac{R}{S} = .905$. Answer: D

15. $\ddot{a}_{x:\overline{10|}} = 1 + vp_x + v^2\,{}_2p_x + \cdots + v^9\,{}_9p_x = 1 + vp + v^2p^2 + \cdots + v^9p^9 = \frac{1-v^{10}p^{10}}{1-vp}$.
$A_{\overset{1}{x}:\overline{10|}} = vq_x + v^2\,{}_{1|}q_x + \cdots + v^{10}\,{}_{9|}q_x = vq + v^2pq + \cdots + v^{10}p^9q$
$= vq[1 + vp + + v^2p^2 + \cdots + v^9p^9] = vq[\frac{1-v^{10}p^{10}}{1-vp}]$. $P_{\overset{1}{x}:\overline{10|}} = \frac{A_{\overset{1}{x}:\overline{10|}}}{\ddot{a}_{x:\overline{10|}}} = vq$.
$vq(1+i) - q = p\,{}_1V \rightarrow 0 = p\,{}_1V \rightarrow {}_1V = 0$.
Same equation continues for $k = 2, 3, \ldots$, so ${}_kV = 0$ for all k. Answer: A

18. $G\,\ddot{a}_{35:\overline{40|}} = .25G + 200 + .1G\,a_{35:\overline{39|}} + 50\,a_{35:\overline{39|}} + 200{,}000\,A_{25:\overline{40|}} + 200\,A_{\overset{1}{25}:\overline{40|}}$
$\rightarrow G = \frac{150 + 50\,\ddot{a}_{25:\overline{40|}} + 200{,}000\,A_{35:\overline{40|}} + 200\,A_{\overset{1}{35}:\overline{40|}}}{.9\,\ddot{a}_{35:\overline{40|}} - .15}$. But $\ddot{a}_{35:\overline{40|}} = \frac{1-A_{35:\overline{40|}}}{d} = 15.4663$,
and $A_{\overset{1}{35}:\overline{40|}} = A_{35:\overline{40|}} - A_{35:\overset{1}{\overline{40|}}} = .047971 \rightarrow G = 1876.78$.
Net premium $= 200{,}000 \times \frac{A_{25:\overline{40|}}}{\ddot{a}_{25:\overline{40|}}} = 1610.58 \rightarrow$ expense loading $= 266.20$. Using the recursive relationship for expense reserves, we have

${}_1V^e = \frac{[266.20 - .25 \times (1876.78) - 200] \times (1.06) - 200\,(.001223)}{1-.001223} = -427.94$, and

${}_2V^e = \frac{[-427.94 + 266.20 - .1 \times (1876.78) - 50] \times (1.06) - 200\,(.0012735)}{1-.0012735} = -424.1$. Answer: B.

19. We use the following general relationship:

$$_mV = \pi \cdot \ddot{s}_{\overline{m}|} - \sum_{k=0}^{m-1} q_{x+k} \cdot (1+i)^{m-k-1} \cdot (b_{k+1} - {}_{k+1}V)$$

For an endowment policy, the reserve at the time of endowment is equal to the endowment amount. In this case, we have $_{30}V = 500,000$. The premium is payable for only 20 years, so in the right hand side of the expression, $\pi \cdot \ddot{s}_{\overline{m}|}$ is replaced by $\pi \cdot \ddot{s}_{\overline{20}|} \cdot (1+i)^{10}$ (this term is the accumulated premium to the time of endowment using interest only). The expression becomes

$$500,000 = \pi \cdot \ddot{s}_{\overline{20}|} \cdot (1+i)^{10} - \sum_{k=0}^{29} q_{35+k} \cdot (1+i)^{30-k-1} \cdot (b_{k+1} - {}_{k+1}V) .$$

Since $b_{k+1} = {}_{k+1}V$ for $k = 0, 1, ..., 28$, and $b_{30} = 0$ and $_{30}V = 500,000$, the RHS becomes

$$500,000 = \pi \cdot \ddot{s}_{\overline{20}|} \cdot (1+i)^{10} - q_{64} \cdot (1+i)^0 \cdot (0 - 500,000)$$

With $i = .04$ and $q_{64} = .02$, solving for π results in $\pi = 10,689$. Answer: E