(e) From the graph, we conclude that the range of f is $\{y \mid y > -1\}$, or the interval $(-1, \infty)$.

(f) The function f is not continuous because there is a "jump" in the graph at $x = 1$.

Now Work PROBLEM 29

EXAMPLE 4 Cost of Electricity

In the summer of 2009, Duke Energy supplied electricity to residences of Ohio for a monthly customer charge of \$4.50 plus 4.2345¢ per kilowatt-hour (kWhr) for the first 1000 kWhr supplied in the month and 5.3622¢ per kWhr for all usage over 1000 kWhr in the month.

(a) What is the charge for using 300 kWhr in a month?

(b) What is the charge for using 1500 kWhr in a month?

(c) If C is the monthly charge for x kWhr, develop a model relating the monthly charge and kilowatt-hours used. That is, express C as a function of x.

Source: *Duke Energy, 2009.*

Solution (a) For 300 kWhr, the charge is \$4.50 plus 4.2345¢ = \$0.042345 per kWhr. That is,

$$\text{Charge} = \$4.50 + \$0.042345(300) = \$17.20$$

(b) For 1500 kWhr, the charge is \$4.50 plus 4.2345¢ per kWhr for the first 1000 kWhr plus 5.3622¢ per kWhr for the 500 kWhr in excess of 1000. That is,

$$\text{Charge} = \$4.50 + \$0.042345(1000) + \$0.053622(500) = \$73.66$$

(c) Let x represent the number of kilowatt-hours used. If $0 \le x \le 1000$, the monthly charge C (in dollars) can be found by multiplying x times \$0.042345 and adding the monthly customer charge of \$4.50. So, if $0 \le x \le 1000$, then $C(x) = 0.042345x + 4.50$.

For $x > 1000$, the charge is $0.042345(1000) + 4.50 + 0.053622(x - 1000)$, since $x - 1000$ equals the usage in excess of 1000 kWhr, which costs \$0.053622 per kWhr. That is, if $x > 1000$, then

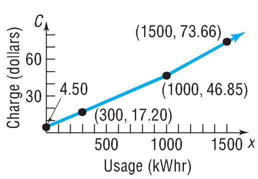
$$\begin{aligned} C(x) &= 0.042345(1000) + 4.50 + 0.053622(x - 1000) \\ &= 46.845 + 0.053622(x - 1000) \\ &= 0.053622x - 6.777 \end{aligned}$$

The rule for computing C follows two equations:

$$C(x) = \begin{cases} 0.042345x + 4.50 & \text{if } 0 \le x \le 1000 \\ 0.053622x - 6.777 & \text{if } x > 1000 \end{cases}$$ The Model

See Figure 42 for the graph.

Figure 42



1.4 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. Sketch the graph of $y = \sqrt{x}$. (p. 15)

2. Sketch the graph of $y = \dfrac{1}{x}$. (pp. 15–16)

3. List the intercepts of the equation $y = x^3 - 8$. (p. 12)

Concepts and Vocabulary

4. The function $f(x) = x^2$ is decreasing on the interval __________.

5. When functions are defined by more than one equation, they are called ________________ functions.

6. ***True or False*** The cube function is odd and is increasing on the interval $(-\infty, \infty)$.

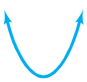
7. ***True or False*** The cube root function is odd and is decreasing on the interval $(-\infty, \infty)$.

8. ***True or False*** The domain and the range of the reciprocal function are the set of all real numbers.

Skill Building

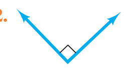
In Problems 9–16, match each graph to its function.

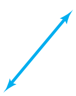
A. Constant function *B. Identity function* *C. Square function*
D. Cube function *E. Square root function* *F. Reciprocal function*
G. Absolute value function *H. Cube root function*

9.


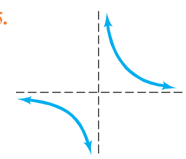
10.


11.


12.


13.


14.


15.


16.


In Problems 17–24, sketch the graph of each function. Be sure to label three points on the graph.

17. $f(x) = x$ **18.** $f(x) = x^2$ **19.** $f(x) = x^3$ **20.** $f(x) = \sqrt{x}$

21. $f(x) = \dfrac{1}{x}$ **22.** $f(x) = |x|$ **23.** $f(x) = \sqrt[3]{x}$ **24.** $f(x) = 3$

25. If $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$

find: (a) $f(-2)$ (b) $f(0)$ (c) $f(2)$

26. If $f(x) = \begin{cases} -3x & \text{if } x < -1 \\ 0 & \text{if } x = -1 \\ 2x^2 + 1 & \text{if } x > -1 \end{cases}$

find: (a) $f(-2)$ (b) $f(-1)$ (c) $f(0)$

27. If $f(x) = \begin{cases} 2x - 4 & \text{if } -1 \le x \le 2 \\ x^3 - 2 & \text{if } 2 < x \le 3 \end{cases}$

find: (a) $f(0)$ (b) $f(1)$ (c) $f(2)$ (d) $f(3)$

28. If $f(x) = \begin{cases} x^3 & \text{if } -2 \le x < 1 \\ 3x + 2 & \text{if } 1 \le x \le 4 \end{cases}$

find: (a) $f(-1)$ (b) $f(0)$ (c) $f(1)$ (d) $f(3)$

In Problems 29–40:

(a) Find the domain of each function. *(b) Locate any intercepts.* *(c) Graph each function.*
(d) Based on the graph, find the range. *(e) Is f continuous on its domain?*

29. $f(x) = \begin{cases} 2x & \text{if } x \ne 0 \\ 1 & \text{if } x = 0 \end{cases}$

30. $f(x) = \begin{cases} 3x & \text{if } x \ne 0 \\ 4 & \text{if } x = 0 \end{cases}$

31. $f(x) = \begin{cases} -2x + 3 & \text{if } x < 1 \\ 3x - 2 & \text{if } x \ge 1 \end{cases}$

32. $f(x) = \begin{cases} x + 3 & \text{if } x < -2 \\ -2x - 3 & \text{if } x \ge -2 \end{cases}$

33. $f(x) = \begin{cases} x + 3 & \text{if } -2 \le x < 1 \\ 5 & \text{if } x = 1 \\ -x + 2 & \text{if } x > 1 \end{cases}$

34. $f(x) = \begin{cases} 2x + 5 & \text{if } -3 \le x < 0 \\ -3 & \text{if } x = 0 \\ -5x & \text{if } x > 0 \end{cases}$

35. $f(x) = \begin{cases} 1 + x & \text{if } x < 0 \\ x^2 & \text{if } x \ge 0 \end{cases}$

36. $f(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 0 \\ \sqrt[3]{x} & \text{if } x \ge 0 \end{cases}$

37. $f(x) = \begin{cases} |x| & \text{if } -2 \le x < 0 \\ x^3 & \text{if } x > 0 \end{cases}$

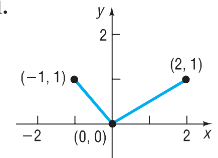
38. $f(x) = \begin{cases} 2 - x & \text{if } -3 \le x < 1 \\ \sqrt{x} & \text{if } x > 1 \end{cases}$

39. $f(x) = 2\,\text{int}(x)$

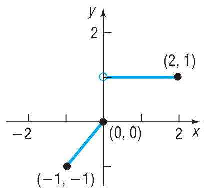
40. $f(x) = \text{int}(2x)$

In Problems 41–44, the graph of a piecewise-defined function is given. Write a definition for each function.

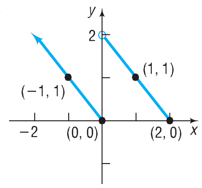
41.



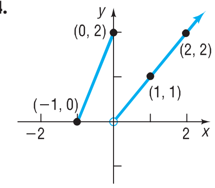
42.



43.



44.



45. If $f(x) = \text{int}(2x)$, find
(a) $f(1.2)$ (b) $f(1.6)$ (c) $f(-1.8)$

46. If $f(x) = \text{int}\left(\frac{x}{2}\right)$, find
(a) $f(1.2)$ (b) $f(1.6)$ (c) $f(-1.8)$

Applications and Extensions

47. Cell Phone Service Sprint PCS offers a monthly cellular phone plan for $39.99. It includes 450 anytime minutes and charges $0.45 per minute for additional minutes. The following function is used to compute the monthly cost for a subscriber:

$$C(x) = \begin{cases} 39.99 & \text{if } 0 \le x \le 450 \\ 0.45x - 162.51 & \text{if } x > 450 \end{cases}$$

where x is the number of anytime minutes used. Compute the monthly cost of the cellular phone for use of the following anytime minutes:
(a) 200 (b) 465 (c) 451

Source: *Sprint PCS*

48. Parking at O'Hare International Airport The short-term (no more than 24 hours) parking fee F (in dollars) for parking x hours at O'Hare International Airport's main parking garage can be modeled by the function

$$F(x) = \begin{cases} 3 & \text{if } 0 < x \le 3 \\ 5\,\text{int}(x + 1) + 1 & \text{if } 3 < x < 9 \\ 50 & \text{if } 9 \le x \le 24 \end{cases}$$

Determine the fee for parking in the short-term parking garage for
(a) 2 hours (b) 7 hours (c) 15 hours
(d) 8 hours and 24 minutes

Source: *O'Hare International Airport*

49. Cost of Natural Gas In April 2009, Peoples Energy had the following rate schedule for natural gas usage in single-family residences:

Monthly service charge	$15.95
Per therm service charge	
1st 50 therms	$0.33606/therm
Over 50 therms	$0.10580/therm
Gas charge	$0.3940/therm

(a) What is the charge for using 50 therms in a month?
(b) What is the charge for using 500 therms in a month?
(c) Develop a model that relates the monthly charge C for x therms of gas.
(d) Graph the function found in part (c).

Source: *Peoples Energy, Chicago, Illinois, 2009*

50. Cost of Natural Gas In April 2009, Nicor Gas had the following rate schedule for natural gas usage in single-family residences:

Monthly customer charge	$8.40
Distribution charge	
1st 20 therms	$0.1473/therm
Next 30 therms	$0.0579/therm
Over 50 therms	$0.0519/therm
Gas supply charge	$0.43/therm

(a) What is the charge for using 40 therms in a month?
(b) What is the charge for using 150 therms in a month?
(c) Develop a model that gives the monthly charge C for x therms of gas.
(d) Graph the function found in part (c).

Source: *Nicor Gas, Aurora, Illinois, 2009*

51. Federal Income Tax Two 2009 Tax Rate Schedules are given in the accompanying table. If x equals taxable income and y equals the tax due, construct a function $y = f(x)$ for Schedule X.

REVISED 2009 TAX RATE SCHEDULES

Schedule X—Single						Schedule Y-1—Married Filing jointly or qualifying Widow(er)					
If Taxable Income Is Over	But Not Over	The Tax Is This Amount		Plus This %	Of the Excess Over	If Taxable Income Is Over	But Not Over	The Tax Is This Amount		Plus This %	Of The Excess Over
$0	$8,350	–	+	10%	$0	$0	$16,700	–	+	10%	$0
8,350	33,950	$835.00	+	15%	8,350	16,700	67,900	$1,670.00	+	15%	16,700
33,950	82,250	4,675.00	+	25%	33,950	67,900	137,050	9,350.00	+	25%	67,900
82,250	171,550	16,750.00	+	28%	82,250	137,050	208,850	26,637.50	+	28%	137,050
171,550	372,950	41,754.00	+	33%	171,550	208,850	372,950	46,741.50	+	33%	208,850
372,950	–	108,216.00	+	35%	372,950	372,950	–	100,894.50	+	35%	372,950

Source: Internal Revenue Service

52. Federal Income Tax Refer to the revised 2009 tax rate schedules. If x equals taxable income and y equals the tax due, construct a function $y = f(x)$ for Schedule Y-1.

53. Cost of Transporting Goods A trucking company transports goods between Chicago and New York, a distance of 960 miles. The company's policy is to charge, for each pound, \$0.50 per mile for the first 100 miles, \$0.40 per mile for the next 300 miles, \$0.25 per mile for the next 400 miles, and no charge for the remaining 160 miles.

(a) Graph the relationship between the cost of transportation in dollars and mileage over the entire 960-mile route.
(b) Find the cost as a function of mileage for hauls between 100 and 400 miles from Chicago.
(c) Find the cost as a function of mileage for hauls between 400 and 800 miles from Chicago.

54. Car Rental Costs An economy car rented in Florida from National Car Rental® on a weekly basis costs \$95 per week. Extra days cost \$24 per day until the day rate exceeds the weekly rate, in which case the weekly rate applies. Also, any part of a day used counts as a full day. Find the cost C of renting an economy car as a function of the number x of days used, where $7 \le x \le 14$. Graph this function.

55. Minimum Payments for Credit Cards Holders of credit cards issued by banks, department stores, oil companies, and so on, receive bills each month that state minimum amounts that must be paid by a certain due date. The minimum due depends on the total amount owed. One such credit card company uses the following rules: For a bill of less than \$10, the entire amount is due. For a bill of at least \$10 but less than \$500, the minimum due is \$10. A minimum of \$30 is due on a bill of at least \$500 but less than \$1000, a minimum of \$50 is due on a bill of at least \$1000 but less than \$1500, and a minimum of \$70 is due on bills of \$1500 or more. Find the function f that describes the minimum payment due on a bill of x dollars. Graph f.

56. Interest Payments for Credit Cards Refer to Problem 55. The card holder may pay any amount between the minimum due and the total owed. The organization issuing the card charges the card holder interest of 1.5% per month for the first \$1000 owed and 1% per month on any unpaid balance over \$1000. Find the function g that gives the amount of interest charged per month on a balance of x dollars. Graph g.

57. Wind Chill The wind chill factor represents the equivalent air temperature at a standard wind speed that would produce the same heat loss as the given temperature and wind speed. One formula for computing the equivalent temperature is

$$W = \begin{cases} t & 0 \le v < 1.79 \\ 33 - \dfrac{(10.45 + 10\sqrt{v} - v)(33 - t)}{22.04} & 1.79 \le v \le 20 \\ 33 - 1.5958(33 - t) & v > 20 \end{cases}$$

where v represents the wind speed (in meters per second) and t represents the air temperature (°C). Compute the wind chill for the following:

(a) An air temperature of 10°C and a wind speed of 1 meter per second (m/sec)
(b) An air temperature of 10°C and a wind speed of 5 m/sec
(c) An air temperature of 10°C and a wind speed of 15 m/sec
(d) An air temperature of 10°C and a wind speed of 25 m/sec
(e) Explain the physical meaning of the equation corresponding to $0 \le v < 1.79$.
(f) Explain the physical meaning of the equation corresponding to $v > 20$.

58. Wind Chill Redo Problem 57(a)–(d) for an air temperature of −10°C.

59. First-class Mail In 2009 the U.S. Postal Service charged \$1.17 postage for first-class mail retail flats (such as an 8.5" by 11" envelope) weighing up to 1 ounce, plus \$0.17 for each additional ounce up to 13 ounces. First-class rates do not apply to flats weighing more than 13 ounces. Develop a model that relates C, the first-class postage charged, for a flat weighing x ounces. Graph the function.

Source: *United States Postal Service*

Explaining Concepts: Discussion and Writing

In Problems 60–67, use a graphing utility.

60. Exploration Graph $y = x^2$. Then on the same screen graph $y = x^2 + 2$, followed by $y = x^2 + 4$, followed by $y = x^2 - 2$. What pattern do you observe? Can you predict the graph of $y = x^2 - 4$? Of $y = x^2 + 5$?

61. Exploration Graph $y = x^2$. Then on the same screen graph $y = (x - 2)^2$, followed by $y = (x - 4)^2$, followed by $y = (x + 2)^2$. What pattern do you observe? Can you predict the graph of $y = (x + 4)^2$? Of $y = (x - 5)^2$?

62. Exploration Graph $y = |x|$. Then on the same screen graph $y = 2|x|$, followed by $y = 4|x|$, followed by $y = \frac{1}{2}|x|$. What pattern do you observe? Can you predict the graph of $y = \frac{1}{4}|x|$? Of $y = 5|x|$?

63. Exploration Graph $y = x^2$. Then on the same screen graph $y = -x^2$. What pattern do you observe? Now try $y = |x|$ and $y = -|x|$. What do you conclude?

64. Exploration Graph $y = \sqrt{x}$. Then on the same screen graph $y = \sqrt{-x}$. What pattern do you observe? Now try $y = 2x + 1$ and $y = 2(-x) + 1$. What do you conclude?

65. Exploration Graph $y = x^3$. Then on the same screen graph $y = (x - 1)^3 + 2$. Could you have predicted the result?

66. Exploration Graph $y = x^2$, $y = x^4$, and $y = x^6$ on the same screen. What do you notice is the same about each graph? What do you notice that is different?

67. Exploration Graph $y = x^3$, $y = x^5$, and $y = x^7$ on the same screen. What do you notice is the same about each graph? What do you notice that is different?

68. Consider the equation

$$y = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$