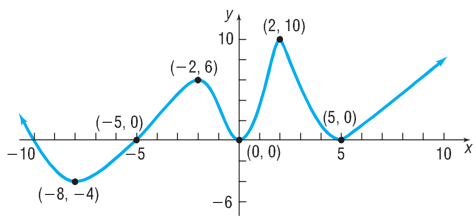
Skill Building

In Problems 11–20, use the given graph of the function f.



11. Is f increasing on the interval $(-8, -2)$?

12. Is f decreasing on the interval $(-8, -4)$?

13. Is f increasing on the interval $(2, 10)$?

14. Is f decreasing on the interval $(2, 5)$?

15. List the interval(s) on which f is increasing.

16. List the interval(s) on which f is decreasing.

17. Is there a local maximum at 2? If yes, what is it?

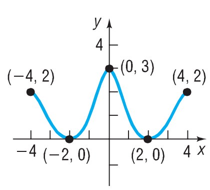
18. Is there a local maximum at 5? If yes, what is it?

19. List the numbers at which f has a local maximum. What are these local maxima?

20. List the numbers at which f has a local minimum. What are these local minima?

In Problems 21–28, the graph of a function is given. Use the graph to find:
(a) The intercepts, if any (b) The domain and range (c) The intervals on which the function is increasing, decreasing, or constant
(d) Whether the function is even, odd, or neither

21.



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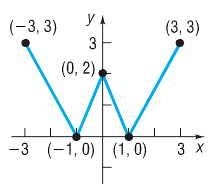

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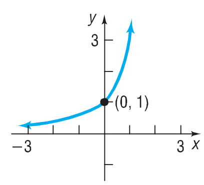

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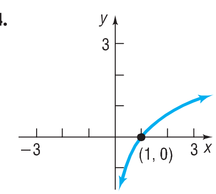

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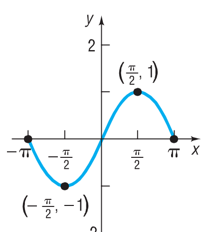

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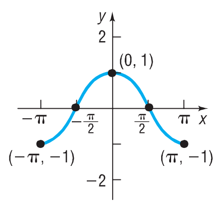

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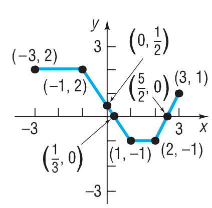

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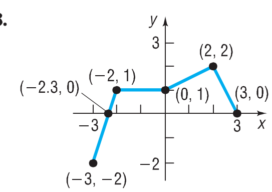
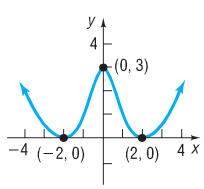


In Problems 29–32, the graph of a function f is given. Use the graph to find:
(a) The values, if any, at which f has a local maximum. What are these local maxima?
(b) The values, if any, at which f has a local minimum. What are these local minima?

29.



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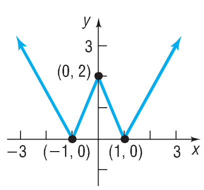

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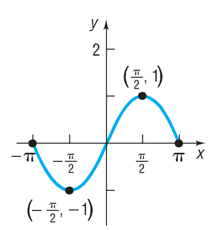

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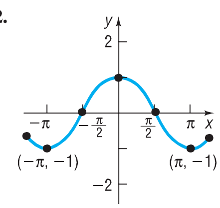
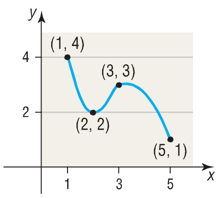


In Problems 33–40, for each graph of a function $y = f(x)$, find the absolute maximum and the absolute minimum, if it exists.

33.



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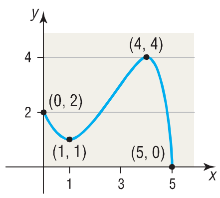

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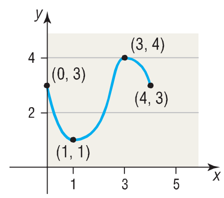

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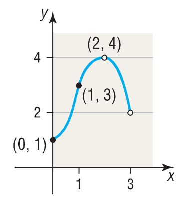

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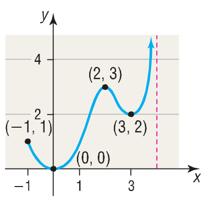

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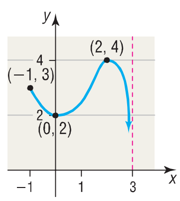

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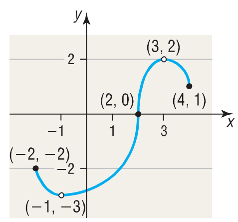

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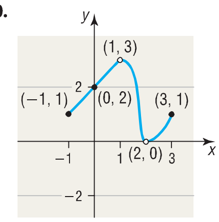


In Problems 41–52, determine algebraically whether each function is even, odd, or neither.

41. $F(x) = 4x^3 + x$

42. $f(x) = 2x^4 - x^2$

43. $g(x) = -3x^2 - 5$

44. $h(x) = 3x^5 + 5x$

45. $F(x) = \sqrt[3]{x}$

46. $G(x) = \sqrt{x}$

47. $f(x) = x + |x|$

48. $f(x) = \sqrt[3]{2x^2 + 1}$

49. $g(x) = \dfrac{1}{x^2}$

50. $h(x) = \dfrac{x}{x^2 - 1}$

51. $h(x) = \dfrac{-x^3}{3x^2 - 9}$

52. $F(x) = \dfrac{2x}{|x|}$

In Problems 53–60, use a graphing utility to graph each function over the indicated interval and approximate any local maxima and local minima. Determine where the function is increasing and where it is decreasing. Round answers to two decimal places.

53. $f(x) = x^3 - 3x + 2 \quad (-2, 2)$

54. $f(x) = x^3 - 3x^2 + 5 \quad (-1, 3)$

55. $f(x) = x^5 - x^3 \quad (-2, 2)$

56. $f(x) = x^4 - x^2 \quad (-2, 2)$

57. $f(x) = -0.2x^3 - 0.6x^2 + 4x - 6 \quad (-6, 4)$

58. $f(x) = -0.4x^3 + 0.6x^2 + 3x - 2 \quad (-4, 5)$

59. $f(x) = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3 \quad (-3, 2)$

60. $f(x) = -0.4x^4 - 0.5x^3 + 0.8x^2 - 2 \quad (-3, 2)$

---

61. Find the average rate of change of $f(x) = -2x^2 + 4$
(a) From 0 to 2
(b) From 1 to 3
(c) From 1 to 4

62. Find the average rate of change of $f(x) = -x^3 + 1$
(a) From 0 to 2
(b) From 1 to 3
(c) From −1 to 1

63. Find the average rate of change of $g(x) = x^3 - 2x + 1$
(a) From −3 to −2
(b) From −1 to 1
(c) From 1 to 3

64. Find the average rate of change of $h(x) = x^2 - 2x + 3$
(a) From −1 to 1
(b) From 0 to 2
(c) From 2 to 5

65. $f(x) = 5x - 2$
(a) Find the average rate of change from 1 to 3.
(b) Find an equation of the secant line containing $(1, f(1))$ and $(3, f(3))$.

66. $f(x) = -4x + 1$
(a) Find the average rate of change from 2 to 5.
(b) Find an equation of the secant line containing $(2, f(2))$ and $(5, f(5))$.

67. $g(x) = x^2 - 2$
(a) Find the average rate of change from −2 to 1.
(b) Find an equation of the secant line containing $(-2, g(-2))$ and $(1, g(1))$.

68. $g(x) = x^2 + 1$
(a) Find the average rate of change from −1 to 2.
(b) Find an equation of the secant line containing $(-1, g(-1))$ and $(2, g(2))$.

69. $h(x) = x^2 - 2x$
(a) Find the average rate of change from 2 to 4.
(b) Find an equation of the secant line containing $(2, h(2))$ and $(4, h(4))$.

70. $h(x) = -2x^2 + x$
(a) Find the average rate of change from 0 to 3.
(b) Find an equation of the secant line containing $(0, h(0))$ and $(3, h(3))$.

Mixed Practice

71. $g(x) = x^3 - 27x$
(a) Determine whether g is even, odd, or neither.
(b) There is a local minimum of -54 at 3. Determine the local maximum.

72. $f(x) = -x^3 + 12x$
(a) Determine whether f is even, odd, or neither.
(b) There is a local maximum of 16 at 2. Determine the local minimum.

73. $F(x) = -x^4 + 8x^2 + 8$
(a) Determine whether F is even, odd, or neither.
(b) There is a local maximum of 24 at $x = 2$. Determine a second local maximum.
(c) Suppose the area under the graph of F between $x = 0$ and $x = 3$ that is bounded below by the x-axis is 47.4 square units. Using the results from part (a), determine the area under the graph of F between $x = -3$ and $x = 0$ bounded below by the x-axis.

74. $G(x) = -x^4 + 32x^2 + 144$
(a) Determine whether G is even, odd, or neither.
(b) There is a local maximum of 400 at $x = 4$. Determine a second local maximum.
(c) Suppose the area under the graph of G between $x = 0$ and $x = 6$ that is bounded below by the x-axis is 1612.8 square units. Using the results from part (a), determine the area under the graph of F between $x = -6$ and $x = 0$ bounded below by the x-axis.

Applications and Extensions

75. Minimum Average Cost The average cost per hour in dollars, $\overline{C}$, of producing x riding lawn mowers can be modeled by the function

$$\overline{C}(x) = 0.3x^2 + 21x - 251 + \frac{2500}{x}$$

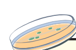
(a) Use a graphing utility to graph $\overline{C} = \overline{C}(x)$.
(b) Determine the number of riding lawn mowers to produce in order to minimize average cost.
(c) What is the minimum average cost?

76. Medicine Concentration The concentration C of a medication in the bloodstream t hours after being administered is modeled by the function

$$C(t) = -0.002x^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085$$

(a) After how many hours will the concentration be highest?
(b) A woman nursing a child must wait until the concentration is below 0.5 before she can feed him. After taking the medication, how long must she wait before feeding her child?

77. E-coli Growth A strain of E-coli Beu 397-recA441 is placed into a nutrient broth at 30° Celsius and allowed to grow. The data shown in the table are collected. The population is measured in grams and the time in hours. Since population P depends on time t and each input corresponds to exactly one output, we can say that population is a function of time; so $P(t)$ represents the population at time t.
(a) Find the average rate of change of the population from 0 to 2.5 hours.
(b) Find the average rate of change of the population from 4.5 to 6 hours.
(c) What is happening to the average rate of change as time passes?



Time (hours), t	Population (grams), P
0	0.09
2.5	0.18
3.5	0.26
4.5	0.35
6	0.50

78. e-Filing Tax Returns The Internal Revenue Service Restructuring and Reform Act (RRA) was signed into law by President Bill Clinton in 1998. A major objective of the RRA was to promote electronic filing of tax returns. The data in the table on the following page show the percentage of individual income tax returns filed electronically for filing years 2000–2008. Since the percentage P of returns filed electronically depends on the filing year y and each input corresponds to exactly one output, the percentage of returns filed electronically is a function of the filing year; so $P(y)$ represents the percentage of returns filed electronically for filing year y.
(a) Find the average rate of change of the percentage of e-filed returns from 2000 to 2002.
(b) Find the average rate of change of the percentage of e-filed returns from 2004 to 2006.
(c) Find the average rate of change of the percentage of e-filed returns from 2006 to 2008.
(d) What is happening to the average rate of change as time passes?

Year	Percentage of returns e-filed
2000	27.9
2001	31.1
2002	35.9
2003	40.6
2004	47.0
2005	51.8
2006	54.5
2007	58.0
2008	59.8

SOURCE: Internal Revenue Service

79. For the function $f(x) = x^2$, compute each average rate of change:
(a) From 0 to 1
(b) From 0 to 0.5
(c) From 0 to 0.1
(d) From 0 to 0.01
(e) From 0 to 0.001
(f) Use a graphing utility to graph each of the secant lines along with f.
(g) What do you think is happening to the secant lines?
(h) What is happening to the slopes of the secant lines? Is there some number that they are getting closer to? What is that number?

80. For the function $f(x) = x^2$, compute each average rate of change:
(a) From 1 to 2
(b) From 1 to 1.5
(c) From 1 to 1.1
(d) From 1 to 1.01
(e) From 1 to 1.001
(f) Use a graphing utility to graph each of the secant lines along with f.
(g) What do you think is happening to the secant lines?
(h) What is happening to the slopes of the secant lines? Is there some number that they are getting closer to? What is that number?

Problems 81–88 require the following discussion of a secant line. The slope of the secant line containing the two points $(x, f(x))$ and $(x + h, f(x + h))$ on the graph of a function $y = f(x)$ may be given as

$$m_{\text{sec}} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h} \qquad h \neq 0$$

In calculus, this expression is called the **difference quotient of *f*.**

(a) Express the slope of the secant line of each function in terms of x and h. Be sure to simplify your answer.
(b) Find m_{sec} for $h = 0.5, 0.1,$ and 0.01 at $x = 1$. What value does m_{sec} approach as h approaches 0?
(c) Find the equation for the secant line at $x = 1$ with $h = 0.01$.
(d) Use a graphing utility to graph f and the secant line found in part (c) on the same viewing window.

81. $f(x) = 2x + 5$
82. $f(x) = -3x + 2$
83. $f(x) = x^2 + 2x$
84. $f(x) = 2x^2 + x$
85. $f(x) = 2x^2 - 3x + 1$
86. $f(x) = -x^2 + 3x - 2$
87. $f(x) = \dfrac{1}{x}$
88. $f(x) = \dfrac{1}{x^2}$

Explaining Concepts: Discussion and Writing

89. Draw the graph of a function that has the following properties: domain: all real numbers; range: all real numbers; intercepts: $(0, -3)$ and $(3, 0)$; a local maximum of -2 is at -1; a local minimum of -6 is at 2. Compare your graph with those of others. Comment on any differences.

90. Redo Problem 89 with the following additional information: increasing on $(-\infty, -1), (2, \infty)$; decreasing on $(-1, 2)$. Again compare your graph with others and comment on any differences.

91. How many x-intercepts can a function defined on an interval have if it is increasing on that interval? Explain.

92. Suppose that a friend of yours does not understand the idea of increasing and decreasing functions. Provide an explanation, complete with graphs, that clarifies the idea.

93. Can a function be both even and odd? Explain.

94. Using a graphing utility, graph $y = 5$ on the interval $(-3, 3)$. Use MAXIMUM to find the local maxima on $(-3, 3)$. Comment on the result provided by the calculator.

95. A function f has a positive average rate of change on the interval [2, 5]. Is f increasing on [2, 5]? Explain.

96. Show that a constant function $f(x) = b$ has an average rate of change of 0. Compute the average rate of change of $y = \sqrt{4 - x^2}$ on the interval $[-2, 2]$. Explain how this can happen.

'Are You Prepared?' Answers

1. $2 < x < 5$ **2.** 1 **3.** symmetric with respect to the y-axis **4.** $y + 2 = 5(x - 3)$ **5.** $(-3, 0), (3, 0), (0, -9)$