CONCEPT PREVIEW *Match each equation in Column I with the correct first step for solving it in Column II.*

I	II
6. $\frac{2x+3}{x}+\frac{5}{x+5}=7$	**A.** Cube each side of the equation.
7. $\sqrt{x+5}=7$	**B.** Multiply each side of the equation by $x(x+5)$.
8. $(x+5)^{5/2}=32$	**C.** Raise each side of the equation to the power $\frac{2}{5}$.
9. $(x+5)^{2/3}-(x+5)^{1/3}-6=0$	**D.** Square each side of the equation.
10. $\sqrt[3]{x(x+5)}=\sqrt[3]{-6}$	**E.** Let $u=(x+5)^{1/3}$ and $u^2=(x+5)^{2/3}$.

Decide what values of the variable cannot possibly be solutions for each equation. Do not solve. ***See Examples 1 and 2.***

11. $\frac{5}{2x+3}-\frac{1}{x-6}=0$

12. $\frac{2}{x+1}+\frac{3}{5x-2}=0$

13. $\frac{3}{x-2}+\frac{1}{x+1}=\frac{3}{x^2-x-2}$

14. $\frac{2}{x+3}-\frac{5}{x-1}=\frac{-5}{x^2+2x-3}$

15. $\frac{1}{4x}-\frac{2}{x}=3$

16. $\frac{5}{2x}+\frac{2}{x}=6$

Solve each equation. ***See Example 1.***

17. $\frac{2x+5}{2}-\frac{3x}{x-2}=x$

18. $\frac{4x+3}{4}-\frac{2x}{x+1}=x$

19. $\frac{x}{x-3}=\frac{3}{x-3}+3$

20. $\frac{x}{x-4}=\frac{4}{x-4}+4$

21. $\frac{-2}{x-3}+\frac{3}{x+3}=\frac{-12}{x^2-9}$

22. $\frac{3}{x-2}+\frac{1}{x+2}=\frac{12}{x^2-4}$

23. $\frac{4}{x^2+x-6}-\frac{1}{x^2-4}=\frac{2}{x^2+5x+6}$

24. $\frac{3}{x^2+x-2}-\frac{1}{x^2-1}=\frac{7}{2x^2+6x+4}$

Solve each equation. ***See Example 2.***

25. $\frac{2x+1}{x-2}+\frac{3}{x}=\frac{-6}{x^2-2x}$

26. $\frac{4x+3}{x+1}+\frac{2}{x}=\frac{1}{x^2+x}$

27. $\frac{x}{x-1}-\frac{1}{x+1}=\frac{2}{x^2-1}$

28. $\frac{-x}{x+1}-\frac{1}{x-1}=\frac{-2}{x^2-1}$

29. $\frac{5}{x^2}-\frac{43}{x}=18$

30. $\frac{7}{x^2}+\frac{19}{x}=6$

31. $2=\frac{3}{2x-1}+\frac{-1}{(2x-1)^2}$

32. $6=\frac{7}{2x-3}+\frac{3}{(2x-3)^2}$

33. $\frac{2x-5}{x}=\frac{x-2}{3}$

34. $\frac{x+4}{2x}=\frac{x-1}{3}$

35. $\frac{2x}{x-2}=5+\frac{4x^2}{x-2}$

36. $\frac{3x^2}{x-1}+2=\frac{x}{x-1}$

Solve each problem. ***See Example 3.***

37. ***Painting a House*** (This problem appears in the 1994 movie *Little Big League*.) If Joe can paint a house in 3 hr, and Sam can paint the same house in 5 hr, how long does it take them to do it together?

38. ***Painting a House*** Repeat **Exercise 37,** but assume that Joe takes 6 hr working alone, and Sam takes 8 hr working alone.

39. ***Pollution in a River*** Two chemical plants are polluting a river. If plant A produces a predetermined maximum amount of pollutant twice as fast as plant B, and together they produce the maximum pollutant in 26 hr, how long will it take plant B alone?

	Rate	Time	Part of Job Completed
Pollution from A	$\frac{1}{x}$	26	$26\left(\frac{1}{x}\right)$
Pollution from B		26	

x represents the number of hours it takes plant A, working alone, to produce the maximum pollutant.

40. ***Filling a Settling Pond*** A sewage treatment plant has two inlet pipes to its settling pond. One pipe can fill the pond 3 times as fast as the other pipe, and together they can fill the pond in 12 hr. How long will it take the faster pipe to fill the pond alone?

	Rate	Time	Part of Job Completed
Faster Inlet Pipe	$\frac{1}{x}$	12	
Slower Inlet Pipe	$\frac{1}{3x}$	12	

x represents the number of hours it takes the faster inlet pipe, working alone, to fill the settling pond.

41. ***Filling a Pool*** An inlet pipe can fill Blake's pool in 5 hr, and an outlet pipe can empty it in 8 hr. In his haste to surf the Internet, Blake left both pipes open. How long did it take to fill the pool?

42. ***Filling a Pool*** Suppose Blake discovered his error (see **Exercise 41**) after an hour-long surf. If he then closed the outlet pipe, how much more time would be needed to fill the pool?

43. ***Filling a Sink*** With both taps open, Robert can fill his kitchen sink in 5 min. When full, the sink drains in 10 min. How long will it take to fill the sink if Robert forgets to put in the stopper?

44. ***Filling a Sink*** If Robert (see **Exercise 43**) remembers to put in the stopper after 1 min, how much longer will it take to fill the sink?

Solve each equation. ***See Examples 4–6.***

45. $x - \sqrt{2x+3} = 0$

46. $x - \sqrt{3x+18} = 0$

47. $\sqrt{3x+7} = 3x+5$

48. $\sqrt{4x+13} = 2x-1$

49. $\sqrt{4x+5} - 6 = 2x - 11$

50. $\sqrt{6x+7} - 9 = x - 7$

51. $\sqrt{4x} - x + 3 = 0$

52. $\sqrt{2x} - x + 4 = 0$

53. $\sqrt{x} - \sqrt{x - 5} = 1$

54. $\sqrt{x} - \sqrt{x - 12} = 2$

55. $\sqrt{x + 7} + 3 = \sqrt{x - 4}$

56. $\sqrt{x + 5} + 2 = \sqrt{x - 1}$

57. $\sqrt{2x + 5} - \sqrt{x + 2} = 1$

58. $\sqrt{4x + 1} - \sqrt{x - 1} = 2$

59. $\sqrt{3x} = \sqrt{5x + 1} - 1$

60. $\sqrt{2x} = \sqrt{3x + 12} - 2$

61. $\sqrt{x + 2} = 1 - \sqrt{3x + 7}$

62. $\sqrt{2x - 5} = 2 + \sqrt{x - 2}$

63. $\sqrt{2\sqrt{7x + 2}} = \sqrt{3x + 2}$

64. $\sqrt{3\sqrt{2x + 3}} = \sqrt{5x - 6}$

65. $3 - \sqrt{x} = \sqrt{2\sqrt{x} - 3}$

66. $\sqrt{x} + 2 = \sqrt{4 + 7\sqrt{x}}$

67. $\sqrt[3]{4x + 3} = \sqrt[3]{2x - 1}$

68. $\sqrt[3]{2x} = \sqrt[3]{5x + 2}$

69. $\sqrt[3]{5x^2 - 6x + 2} - \sqrt[3]{x} = 0$

70. $\sqrt[3]{3x^2 - 9x + 8} = \sqrt[3]{x}$

71. $\sqrt[4]{x - 15} = 2$

72. $\sqrt[4]{3x + 1} = 1$

73. $\sqrt[4]{x^2 + 2x} = \sqrt[4]{3}$

74. $\sqrt[4]{x^2 + 6x} = 2$

Solve each equation. ***See Example 7.***

75. $x^{3/2} = 125$

76. $x^{5/4} = 32$

77. $(x^2 + 24x)^{1/4} = 3$

78. $(3x^2 + 52x)^{1/4} = 4$

79. $(x - 3)^{2/5} = 4$

80. $(x + 200)^{2/3} = 36$

81. $(2x + 5)^{1/3} - (6x - 1)^{1/3} = 0$

82. $(3x + 7)^{1/3} - (4x + 2)^{1/3} = 0$

83. $(2x - 1)^{2/3} = x^{1/3}$

84. $(x - 3)^{2/5} = (4x)^{1/5}$

85. $x^{2/3} = 2x^{1/3}$

86. $3x^{3/4} = x^{1/2}$

Solve each equation. ***See Examples 8 and 9.***

87. $2x^4 - 7x^2 + 5 = 0$

88. $4x^4 - 8x^2 + 3 = 0$

89. $x^4 + 2x^2 - 15 = 0$

90. $3x^4 + 10x^2 - 25 = 0$

91. $(x - 1)^{2/3} + (x - 1)^{1/3} - 12 = 0$

92. $(2x - 1)^{2/3} + 2(2x - 1)^{1/3} - 3 = 0$

93. $(x + 1)^{2/5} - 3(x + 1)^{1/5} + 2 = 0$

94. $(x + 5)^{2/3} + (x + 5)^{1/3} - 20 = 0$

95. $4(x + 1)^4 - 13(x + 1)^2 = -9$

96. $25(x - 5)^4 - 116(x - 5)^2 = -64$

97. $6(x + 2)^4 - 11(x + 2)^2 = -4$

98. $8(x - 4)^4 - 10(x - 4)^2 = -3$

99. $10x^{-2} + 33x^{-1} - 7 = 0$

100. $7x^{-2} - 10x^{-1} - 8 = 0$

101. $x^{-2/3} + x^{-1/3} - 6 = 0$

102. $2x^{-2/5} - x^{-1/5} - 1 = 0$

103. $16x^{-4} - 65x^{-2} + 4 = 0$

104. $625x^{-4} - 125x^{-2} + 4 = 0$

1.6 Other Types of Equations and Applications

- Rational Equations
- Work Rate Problems
- Equations with Radicals
- Equations with Rational Exponents
- Equations Quadratic in Form

Rational Equations

A **rational equation** is an equation that has a rational expression for one or more terms. To solve a rational equation, multiply each side by the least common denominator (LCD) of the terms of the equation to eliminate fractions, and then solve the resulting equation.

A value of the variable that appears to be a solution after each side of a rational equation is multiplied by a variable expression (the LCD) is called a **proposed solution.** Because a rational expression is not defined when its denominator is 0, **proposed solutions for which any denominator equals 0 are excluded from the solution set.**

Be sure to check all proposed solutions in the original equation.

EXAMPLE 1 Solving Rational Equations That Lead to Linear Equations

Solve each equation.

(a) $\frac{3x-1}{3} - \frac{2x}{x-1} = x$ **(b)** $\frac{x}{x-2} = \frac{2}{x-2} + 2$

SOLUTION

(a) The least common denominator is $3(x-1)$, which is equal to 0 if $x = 1$. Therefore, 1 cannot possibly be a solution of this equation.

$$\frac{3x-1}{3} - \frac{2x}{x-1} = x$$

$$3(x-1)\left(\frac{3x-1}{3}\right) - 3(x-1)\left(\frac{2x}{x-1}\right) = 3(x-1)x \quad \text{Multiply by the LCD, } 3(x-1)\text{, where } x \neq 1.$$

$$(x-1)(3x-1) - 3(2x) = 3x(x-1) \quad \text{Divide out common factors.}$$

$$3x^2 - 4x + 1 - 6x = 3x^2 - 3x \quad \text{Multiply.}$$

$$1 - 10x = -3x \quad \text{Subtract } 3x^2\text{. Combine like terms.}$$

$$1 = 7x \quad \text{Solve the linear equation.}$$

$$x = \frac{1}{7} \quad \text{Proposed solution}$$

The proposed solution $\frac{1}{7}$ meets the requirement that $x \neq 1$ and does not cause any denominator to equal 0. Substitute to check for correct algebra.

CHECK

$$\frac{3x-1}{3} - \frac{2x}{x-1} = x \quad \text{Original equation}$$

$$\frac{3\left(\frac{1}{7}\right)-1}{3} - \frac{2\left(\frac{1}{7}\right)}{\frac{1}{7}-1} \stackrel{?}{=} \frac{1}{7} \quad \text{Let } x = \tfrac{1}{7}.$$

$$-\frac{4}{21} - \left(-\frac{1}{3}\right) \stackrel{?}{=} \frac{1}{7} \quad \text{Simplify the complex fractions.}$$

$$\frac{1}{7} = \frac{1}{7} \checkmark \quad \text{True}$$

The solution set is $\left\{\frac{1}{7}\right\}$.

(b)
$$\frac{x}{x-2} = \frac{2}{x-2} + 2$$

$$(x-2)\left(\frac{x}{x-2}\right) = (x-2)\left(\frac{2}{x-2}\right) + (x-2)2$$ Multiply by the LCD, $x-2$, where $x \neq 2$.

$$x = 2 + 2(x-2)$$ Divide out common factors.

$$x = 2 + 2x - 4$$ Distributive property

$$-x = -2$$ Solve the linear equation.

$$x = 2$$ Proposed solution

The proposed solution is 2. However, the variable is restricted to real numbers except 2. If $x = 2$, then not only does it cause a zero denominator, but also multiplying by $x - 2$ in the first step is multiplying both sides by 0, which is not valid. Thus, the solution set is $\emptyset$.

✔ Now Try Exercises 17 and 19.

EXAMPLE 2 Solving Rational Equations That Lead to Quadratic Equations

Solve each equation.

(a) $\dfrac{3x+2}{x-2} + \dfrac{1}{x} = \dfrac{-2}{x^2-2x}$ **(b)** $\dfrac{-4x}{x-1} + \dfrac{4}{x+1} = \dfrac{-8}{x^2-1}$

SOLUTION

(a)
$$\frac{3x+2}{x-2} + \frac{1}{x} = \frac{-2}{x^2-2x}$$

$$\frac{3x+2}{x-2} + \frac{1}{x} = \frac{-2}{x(x-2)}$$ Factor the last denominator.

$$x(x-2)\left(\frac{3x+2}{x-2}\right) + x(x-2)\left(\frac{1}{x}\right) = x(x-2)\left(\frac{-2}{x(x-2)}\right)$$ Multiply by $x(x-2)$, $x \neq 0, 2$.

$$x(3x+2) + (x-2) = -2$$ Divide out common factors.

$$3x^2 + 2x + x - 2 = -2$$ Distributive property

$$3x^2 + 3x = 0$$ Standard form

$$3x(x+1) = 0$$ Factor.

Set *each* factor equal to 0. $$3x = 0 \quad \text{or} \quad x + 1 = 0$$ Zero-factor property

$$x = 0 \quad \text{or} \quad x = -1$$ Proposed solutions

Because of the restriction $x \neq 0$, the only valid proposed solution is -1. Check -1 in the original equation. The solution set is $\{-1\}$.

(b)
$$\frac{-4x}{x-1} + \frac{4}{x+1} = \frac{-8}{x^2-1}$$

$$\frac{-4x}{x-1} + \frac{4}{x+1} = \frac{-8}{(x+1)(x-1)}$$ Factor.

The restrictions on x are $x \neq \pm 1$. Multiply by the LCD, $(x+1)(x-1)$.

$$(x+1)(x-1)\left(\frac{-4x}{x-1}\right)+(x+1)(x-1)\left(\frac{4}{x+1}\right)=(x+1)(x-1)\left(\frac{-8}{(x+1)(x-1)}\right)$$

$$-4x(x+1)+4(x-1)=-8 \quad \text{Divide out common factors.}$$

$$-4x^2-4x+4x-4=-8 \quad \text{Distributive property}$$

$$-4x^2+4=0 \quad \text{Standard form}$$

$$x^2-1=0 \quad \text{Divide by } -4.$$

$$(x+1)(x-1)=0 \quad \text{Factor.}$$

$$x+1=0 \quad \text{or} \quad x-1=0 \quad \text{Zero-factor property}$$

$$x=-1 \quad \text{or} \quad x=1 \quad \text{Proposed solutions}$$

Neither proposed solution is valid, so the solution set is $\emptyset$.

✔ Now Try Exercises 25 and 27.

Work Rate Problems If a job can be completed in 3 hr, then the rate of work is $\frac{1}{3}$ of the job per hr. After 1 hr the job would be $\frac{1}{3}$ complete, and after 2 hr the job would be $\frac{2}{3}$ complete. In 3 hr the job would be $\frac{3}{3}$ complete, meaning that 1 complete job had been accomplished.

PROBLEM-SOLVING HINT If a job can be completed in t units of time, then the rate of work, r, is $\frac{1}{t}$ of the job per unit time.

$$r=\frac{1}{t}$$

The amount of work completed, A, is found by multiplying the rate of work, r, and the amount of time worked, t. This formula is similar to the distance formula $d = rt$.

Amount of work completed = rate of work × amount of time worked

or $$A = rt$$

EXAMPLE 3 Solving a Work Rate Problem

One printer can do a job twice as fast as another. Working together, both printers can do the job in 2 hr. How long would it take each printer, working alone, to do the job?

SOLUTION

Step 1 **Read** the problem. We must find the time it would take each printer, working alone, to do the job.

Step 2 **Assign a variable.** Let x represent the number of hours it would take the faster printer, working alone, to do the job. The time for the slower printer to do the job alone is then $2x$ hours.

Therefore, $\frac{1}{x}$ = the rate of the faster printer (job per hour)

and $\frac{1}{2x}$ = the rate of the slower printer (job per hour).

The time for the printers to do the job together is 2 hr. Multiplying each rate by the time will give the fractional part of the job completed by each.

	Rate	Time	Part of the Job Completed
Faster Printer	$\frac{1}{x}$	2	$2\left(\frac{1}{x}\right) = \frac{2}{x}$
Slower Printer	$\frac{1}{2x}$	2	$2\left(\frac{1}{2x}\right) = \frac{1}{x}$

$A = rt$

Step 3 **Write an equation.** The sum of the two parts of the job completed is 1 because one whole job is done.

$$\underbrace{\frac{2}{x}}_{\substack{\text{Part of the job}\\\text{done by the}\\\text{faster printer}}} + \underbrace{\frac{1}{x}}_{\substack{\text{Part of the job}\\\text{done by the}\\\text{slower printer}}} = \underbrace{1}_{\substack{\text{One whole}\\\text{job}}}$$

Step 4 **Solve.**

$$x\left(\frac{2}{x} + \frac{1}{x}\right) = x(1) \quad \text{Multiply each side by } x\text{, where } x \neq 0.$$

$$x\left(\frac{2}{x}\right) + x\left(\frac{1}{x}\right) = x(1) \quad \text{Distributive property}$$

$$2 + 1 = x \quad \text{Multiply.}$$

$$3 = x \quad \text{Add.}$$

Step 5 **State the answer.** The faster printer would take 3 hr to do the job alone. The slower printer would take $2(3) = 6$ hr. Give *both* answers here.

Step 6 **Check.** The answer is reasonable because the time working together (2 hr, as stated in the problem) is less than the time it would take the faster printer working alone (3 hr, as found in Step 4).

Now Try Exercise 39.

NOTE **Example 3** can also be solved by using the fact that the sum of the rates of the individual printers is equal to their rate working together. Because the printers can complete the job together in 2 hr, their combined rate is $\frac{1}{2}$ of the job per hr.

$$\frac{1}{x} + \frac{1}{2x} = \frac{1}{2}$$

$$2x\left(\frac{1}{x} + \frac{1}{2x}\right) = 2x\left(\frac{1}{2}\right) \quad \text{Multiply each side by } 2x.$$

$$2 + 1 = x \quad \text{Distributive property}$$

$$3 = x \quad \text{Same solution found earlier}$$

Equations with Radicals To solve an equation such as

$$x - \sqrt{15 - 2x} = 0,$$

in which the variable appears in a radicand, we use the following **power property** to eliminate the radical.

Power Property

If P and Q are algebraic expressions, then every solution of the equation $P = Q$ is also a solution of the equation $P^n = Q^n$, for any positive integer n.

When the power property is used to solve equations, the new equation may have *more* solutions than the original equation. For example, the equation

$$x = -2 \quad \text{has solution set} \quad \{-2\}.$$

If we square each side of the equation $x = -2$, we obtain the new equation

$$x^2 = 4, \quad \text{which has solution set} \quad \{-2, 2\}.$$

Because the solution sets are not equal, the equations are not equivalent. ***When we use the power property to solve an equation, it is essential to check all proposed solutions in the original equation.***

CAUTION ***Be very careful when using the power property.*** It does *not* say that the equations $P = Q$ and $P^n = Q^n$ are equivalent. It says only that each solution of the original equation $P = Q$ is also a solution of the new equation $P^n = Q^n$.

Solving an Equation Involving Radicals

Step 1 Isolate the radical on one side of the equation.

Step 2 Raise each side of the equation to a power that is the same as the index of the radical so that the radical is eliminated.

If the equation still contains a radical, repeat Steps 1 and 2.

Step 3 Solve the resulting equation.

Step 4 Check each proposed solution in the *original* equation.

EXAMPLE 4 Solving an Equation Containing a Radical (Square Root)

Solve $x - \sqrt{15 - 2x} = 0$.

SOLUTION

$$x - \sqrt{15 - 2x} = 0$$

Step 1 $\quad x = \sqrt{15 - 2x} \quad$ Isolate the radical.

Step 2 $\quad x^2 = \left(\sqrt{15 - 2x}\right)^2 \quad$ Square each side.

$\quad x^2 = 15 - 2x \quad$ $\left(\sqrt{a}\right)^2 = a$, for $a \ge 0$.

Step 3 $\quad x^2 + 2x - 15 = 0 \quad$ Write in standard form.

$\quad (x + 5)(x - 3) = 0 \quad$ Factor.

$\quad x + 5 = 0 \quad \text{or} \quad x - 3 = 0 \quad$ Zero-factor property

$\quad x = -5 \quad \text{or} \quad x = 3 \quad$ Proposed solutions

Step 4

CHECK $\quad x - \sqrt{15 - 2x} = 0 \quad$ Original equation

Let $x = -5$.		Let $x = 3$.	
$-5 - \sqrt{15 - 2(-5)} \stackrel{?}{=} 0$	Let $x = -5$.	$3 - \sqrt{15 - 2(3)} \stackrel{?}{=} 0$	Let $x = 3$.
$-5 - \sqrt{25} \stackrel{?}{=} 0$		$3 - \sqrt{9} \stackrel{?}{=} 0$	
$-5 - 5 \stackrel{?}{=} 0$		$3 - 3 \stackrel{?}{=} 0$	
$-10 = 0$	False	$0 = 0$ ✓	True

As the check shows, only 3 is a solution, so the solution set is $\{3\}$.

✔ Now Try Exercise 45.

EXAMPLE 5 Solving an Equation Containing Two Radicals

Solve $\sqrt{2x + 3} - \sqrt{x + 1} = 1$.

SOLUTION

$$\sqrt{2x + 3} - \sqrt{x + 1} = 1$$ Isolate one of the radicals on one side of the equation.

Step 1 $\quad \sqrt{2x + 3} = 1 + \sqrt{x + 1} \quad$ Isolate $\sqrt{2x + 3}$.

Step 2 $\quad \left(\sqrt{2x + 3}\right)^2 = \left(1 + \sqrt{x + 1}\right)^2 \quad$ Square each side.

$$2x + 3 = 1 + 2\sqrt{x + 1} + (x + 1)$$ **Be careful:** $(a + b)^2 = a^2 + 2ab + b^2$

Don't forget this term when squaring.

Step 1 $\quad x + 1 = 2\sqrt{x + 1} \quad$ Isolate the remaining radical.

Step 2 $\quad (x + 1)^2 = \left(2\sqrt{x + 1}\right)^2 \quad$ Square again.

$$x^2 + 2x + 1 = 4(x + 1)$$ $(ab)^2 = a^2b^2$ $\quad$ Apply the exponents.

$$x^2 + 2x + 1 = 4x + 4$$ Distributive property

Step 3 $\quad x^2 - 2x - 3 = 0 \quad$ Write in standard form.

$$(x - 3)(x + 1) = 0$$ Factor.

$$x - 3 = 0 \quad \text{or} \quad x + 1 = 0$$ Zero-factor property

$$x = 3 \quad \text{or} \quad x = -1$$ Proposed solutions

Step 4

CHECK $\quad \sqrt{2x + 3} - \sqrt{x + 1} = 1 \quad$ Original equation

Let $x = 3$.		Let $x = -1$.	
$\sqrt{2(3) + 3} - \sqrt{3 + 1} \stackrel{?}{=} 1$	Let $x = 3$.	$\sqrt{2(-1) + 3} - \sqrt{-1 + 1} \stackrel{?}{=} 1$	Let $x = -1$.
$\sqrt{9} - \sqrt{4} \stackrel{?}{=} 1$		$\sqrt{1} - \sqrt{0} \stackrel{?}{=} 1$	
$3 - 2 \stackrel{?}{=} 1$		$1 - 0 \stackrel{?}{=} 1$	
$1 = 1$ ✓	True	$1 = 1$ ✓	True

Both 3 and -1 are solutions of the original equation, so $\{-1, 3\}$ is the solution set.

✔ Now Try Exercise 57.

CAUTION Remember to isolate a radical in Step 1. It would be incorrect to square each term individually as the first step in **Example 5.**

EXAMPLE 6 Solving an Equation Containing a Radical (Cube Root)

Solve $\sqrt[3]{4x^2 - 4x + 1} - \sqrt[3]{x} = 0$.

SOLUTION

$$\sqrt[3]{4x^2 - 4x + 1} - \sqrt[3]{x} = 0$$

Step 1 $\sqrt[3]{4x^2 - 4x + 1} = \sqrt[3]{x}$ Isolate a radical.

Step 2 $\left(\sqrt[3]{4x^2 - 4x + 1}\right)^3 = \left(\sqrt[3]{x}\right)^3$ Cube each side.

$4x^2 - 4x + 1 = x$ Apply the exponents.

Step 3 $4x^2 - 5x + 1 = 0$ Write in standard form.

$(4x - 1)(x - 1) = 0$ Factor.

$4x - 1 = 0$ or $x - 1 = 0$ Zero-factor property

$x = \frac{1}{4}$ or $x = 1$ Proposed solutions

Step 4

CHECK $\sqrt[3]{4x^2 - 4x + 1} - \sqrt[3]{x} = 0$ Original equation

$\sqrt[3]{4\left(\frac{1}{4}\right)^2 - 4\left(\frac{1}{4}\right) + 1} - \sqrt[3]{\frac{1}{4}} \stackrel{?}{=} 0$ Let $x = \frac{1}{4}$.

$\sqrt[3]{\frac{1}{4}} - \sqrt[3]{\frac{1}{4}} \stackrel{?}{=} 0$

$0 = 0$ ✓ True

$\sqrt[3]{4(1)^2 - 4(1) + 1} - \sqrt[3]{1} \stackrel{?}{=} 0$ Let $x = 1$.

$\sqrt[3]{1} - \sqrt[3]{1} \stackrel{?}{=} 0$

$0 = 0$ ✓ True

Both are valid solutions, and the solution set is $\left\{\frac{1}{4}, 1\right\}$.

✔ Now Try Exercise 69.

Equations with Rational Exponents An equation with a rational exponent contains a variable, or variable expression, raised to an exponent that is a rational number. For example, the radical equation

$$\left(\sqrt[5]{x}\right)^3 = 27 \quad \text{can be written with a rational exponent as} \quad x^{3/5} = 27$$

and solved by raising each side to the reciprocal of the exponent, with care taken regarding signs as seen in **Example 7(b).**

EXAMPLE 7 Solving Equations with Rational Exponents

Solve each equation.

(a) $x^{3/5} = 27$ **(b)** $(x - 4)^{2/3} = 16$

SOLUTION

(a) $x^{3/5} = 27$

$\left(x^{3/5}\right)^{5/3} = 27^{5/3}$ Raise each side to the power $\frac{5}{3}$, the reciprocal of the exponent of x.

$x = 243$ $27^{5/3} = \left(\sqrt[3]{27}\right)^5 = 3^5 = 243$