Solve each problem. ***See Examples 1 and 2.***

19. ***Dimensions of a Right Triangle*** The lengths of the sides of a right triangle are consecutive even integers. Find these lengths. (*Hint:* Use the Pythagorean theorem.)

20. ***Dimensions of a Right Triangle*** The lengths of the sides of a right triangle are consecutive positive integers. Find these lengths. (*Hint:* Use the Pythagorean theorem.)

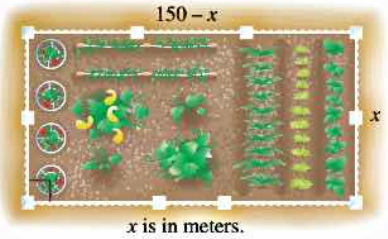
21. ***Dimensions of a Square*** The length of each side of a square is 3 in. more than the length of each side of a smaller square. The sum of the areas of the squares is 149 in.^{2}. Find the lengths of the sides of the two squares.

22. ***Dimensions of a Square*** The length of each side of a square is 5 in. more than the length of each side of a smaller square. The difference of the areas of the squares is 95 in.^{2}. Find the lengths of the sides of the two squares.

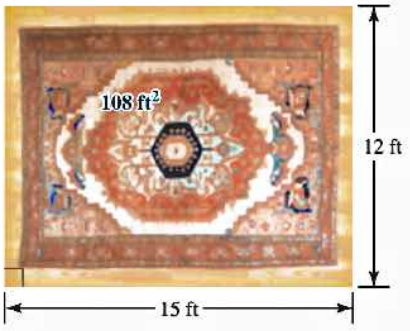
Solve each problem. ***See Example 1.***

23. ***Dimensions of a Parking Lot*** A parking lot has a rectangular area of 40,000 yd^2. The length is 200 yd more than twice the width. Find the dimensions of the lot.

24. ***Dimensions of a Garden*** An ecology center wants to set up an experimental garden using 300 m of fencing to enclose a rectangular area of 5000 m^2. Find the dimensions of the garden.



25. ***Dimensions of a Rug*** Zachary wants to buy a rug for a room that is 12 ft wide and 15 ft long. He wants to leave a uniform strip of floor around the rug. He can afford to buy 108 ft^2 of carpeting. What dimensions should the rug have?



26. ***Width of a Flower Border*** A landscape architect has included a rectangular flower bed measuring 9 ft by 5 ft in her plans for a new building. She wants to use two colors of flowers in the bed: one in the center and the other for a border of the same width on all four sides. If she has enough plants to cover 24 ft^2 for the border, how wide can the border be?

27. ***Volume of a Box*** A rectangular piece of metal is 10 in. longer than it is wide. Squares with sides 2 in. long are cut from the four corners, and the flaps are folded upward to form an open box. If the volume of the box is 832 in.^{3}, what were the original dimensions of the piece of metal?

28. ***Volume of a Box*** In **Exercise 27,** suppose that the piece of metal has length twice the width, and 4-in. squares are cut from the corners. If the volume of the box is 1536 in.^{3}, what were the original dimensions of the piece of metal?

29. ***Manufacturing to Specifications*** A manufacturing firm wants to package its product in a cylindrical container 3 ft high with surface area 8π ft^2. What should the radius of the circular top and bottom of the container be? (*Hint:* The surface area consists of the circular top and bottom and a rectangle that represents the side cut open vertically and unrolled.)

30. ***Manufacturing to Specifications*** In **Exercise 29,** what radius would produce a container with a volume of π times the radius? (*Hint:* The volume is the area of the circular base times the height.)

31. ***Dimensions of a Square*** What is the length of the side of a square if its area and perimeter are numerically equal?

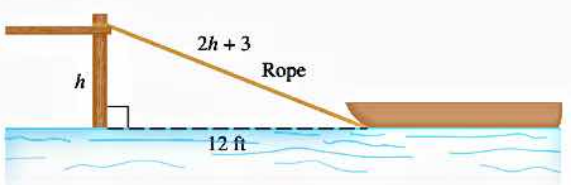
32. ***Dimensions of a Rectangle*** A rectangle has an area that is numerically twice its perimeter. If the length is twice the width, what are its dimensions?

33. ***Radius of a Can*** A can of Blue Runner Red Kidney Beans has surface area 371 cm^2. Its height is 12 cm. What is the radius of the circular top? Round to the nearest hundredth.

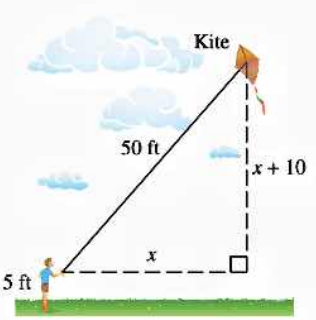
34. ***Dimensions of a Cereal Box*** The volume of a 15-oz cereal box is 180.4 in.^{3}. The length of the box is 3.2 in. less than the height, and its width is 2.3 in. Find the height and length of the box to the nearest tenth.

Solve each problem. ***See Example 2.***

35. ***Height of a Dock*** A boat is being pulled into a dock with a rope attached to the boat at water level. When the boat is 12 ft from the dock, the length of the rope from the boat to the dock is 3 ft longer than twice the height of the dock above the water. Find the height of the dock.



36. ***Height of a Kite*** Grady is flying a kite on 50 ft of string. Its vertical distance from his hand is 10 ft more than its horizontal distance from his hand. Assuming that the string is being held 5 ft above ground level, find its horizontal distance from Grady and its vertical distance from the ground.



1.5 Applications and Modeling with Quadratic Equations

- Geometry Problems
- The Pythagorean Theorem
- Height of a Projected Object
- Modeling with Quadratic Equations

Geometry Problems

To solve these applications, we continue to use a six-step problem-solving strategy.

EXAMPLE 1 Solving a Problem Involving Volume

A piece of machinery produces rectangular sheets of metal such that the length is three times the width. Equal-sized squares measuring 5 in. on a side can be cut from the corners so that the resulting piece of metal can be shaped into an open box by folding up the flaps. If specifications call for the volume of the box to be 1435 in.^{3}, find the dimensions of the original piece of metal.

SOLUTION

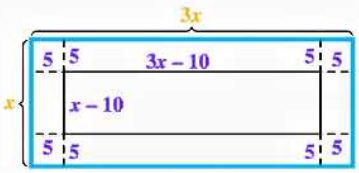


Figure 5

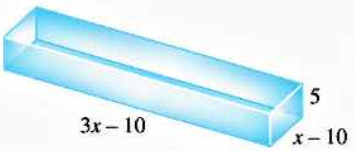


Figure 6

Step 1 **Read** the problem. We must find the dimensions of the original piece of metal.

Step 2 **Assign a variable.** We know that the length is three times the width.

Let x = the width (in inches) and thus, $3x$ = the length.

The box is formed by cutting $5 + 5 = 10$ in. from both the length and the width. See **Figure 5.** The width of the bottom of the box is $x - 10$, the length of the bottom of the box is $3x - 10$, and the height is 5 in. (the length of the side of each cut-out square). See **Figure 6.**

Step 3 **Write an equation.** The formula for volume of a box is $V = lwh$.

$$\begin{array}{ccc} \text{Volume} & = & \text{length} \times \text{width} \times \text{height} \\ \downarrow & & \downarrow \quad \downarrow \quad \downarrow \\ 1435 & = & (3x - 10)(x - 10)(5) \end{array}$$

(Note that the dimensions of the box must be positive numbers, so $3x - 10$ and $x - 10$ must be greater than 0, which implies $x > \frac{10}{3}$ and $x > 10$. These are both satisfied when $x > 10$.)

Step 4 **Solve** the equation from Step 3.

$$1435 = 15x^2 - 200x + 500 \qquad \text{Multiply.}$$

$$0 = 15x^2 - 200x - 935 \qquad \text{Subtract 1435 from each side.}$$

$$0 = 3x^2 - 40x - 187 \qquad \text{Divide each side by 5.}$$

$$0 = (3x + 11)(x - 17) \qquad \text{Factor.}$$

$$3x + 11 = 0 \quad \text{or} \quad x - 17 = 0 \qquad \text{Zero-factor property}$$

The width cannot be negative. $x = -\frac{11}{3}$ or $x = 17$ Solve each equation.

Step 5 **State the answer.** Only 17 satisfies the restriction $x > 10$. Thus, the dimensions of the original piece should be 17 in. by $3(17) = 51$ in.

Step 6 **Check.** The length and width of the bottom of the box are

$$51 - 2(5) = 41 \text{ in.} \qquad \text{Length}$$

and

$$17 - 2(5) = 7 \text{ in.} \qquad \text{Width}$$

The height is 5 in. (the amount cut on each corner), so the volume is

$$V = lwh = 41 \times 7 \times 5 = 1435 \text{ in.}^3, \quad \text{as required.}$$

Now Try Exercise 27.

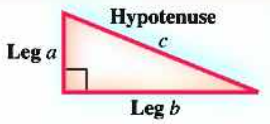
PROBLEM-SOLVING HINT As seen in **Example 1,** discard any solution that does not satisfy the physical constraints of a problem.

The Pythagorean Theorem **Example 2** requires the use of the **Pythagorean theorem** for right triangles. Recall that the **legs** of a right triangle form the right angle, and the **hypotenuse** is the side opposite the right angle.

Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$a^2 + b^2 = c^2$$



EXAMPLE 2 Applying the Pythagorean Theorem

A piece of property has the shape of a right triangle. The longer leg is 20 m longer than twice the length of the shorter leg. The hypotenuse is 10 m longer than the length of the longer leg. Find the lengths of the sides of the triangular lot.

SOLUTION

Step 1 **Read** the problem. We must find the lengths of the three sides.

Step 2 **Assign a variable.**

Let x = the length of the shorter leg (in meters).

Then $2x + 20$ = the length of the longer leg, and

$(2x + 20) + 10$, or $2x + 30$ = the length of the hypotenuse.

See **Figure 7.**

x is in meters.

Figure 7

Step 3 **Write an equation.**

$$\underset{\downarrow}{a^2} + \underset{\downarrow}{b^2} = \underset{\downarrow}{c^2}$$

The hypotenuse is c.

$$x^2 + (2x + 20)^2 = (2x + 30)^2$$

Substitute into the Pythagorean theorem.

Step 4 **Solve** the equation.

$$x^2 + (4x^2 + 80x + 400) = 4x^2 + 120x + 900$$

Square the binomials. Remember the middle terms.

$$x^2 - 40x - 500 = 0$$

Standard form

$$(x - 50)(x + 10) = 0$$

Factor.

$$x - 50 = 0 \quad \text{or} \quad x + 10 = 0$$

Zero-factor property

$$x = 50 \quad \text{or} \quad x = -10$$

Solve each equation.

Step 5 **State the answer.** Because x represents a length, -10 is not reasonable. The lengths of the sides of the triangular lot are

$$50 \text{ m}, \quad 2(50) + 20 = 120 \text{ m}, \quad \text{and} \quad 2(50) + 30 = 130 \text{ m}.$$

Step 6 **Check.** The lengths 50, 120, and 130 satisfy the words of the problem and also satisfy the Pythagorean theorem.

Now Try Exercise 35.