1.4 Exercises

CONCEPT PREVIEW *Match the equation in Column I with its solution(s) in Column II.*

I		II	
1. $x^2 = 25$	**2.** $x^2 = -25$	**A.** $\pm 5i$	**B.** $\pm 2\sqrt{5}$
3. $x^2 + 5 = 0$	**4.** $x^2 - 5 = 0$	**C.** $\pm i\sqrt{5}$	**D.** 5
5. $x^2 = -20$	**6.** $x^2 = 20$	**E.** $\pm\sqrt{5}$	**F.** -5
7. $x - 5 = 0$	**8.** $x + 5 = 0$	**G.** ± 5	**H.** $\pm 2i\sqrt{5}$

CONCEPT PREVIEW *Use Choices A–D to answer each question.*

A. $3x^2 - 17x - 6 = 0$ **B.** $(2x + 5)^2 = 7$

C. $x^2 + x = 12$ **D.** $(3x - 1)(x - 7) = 0$

9. Which equation is set up for direct use of the zero-factor property? Solve it.

10. Which equation is set up for direct use of the square root property? Solve it.

11. Only one of the equations does not require Step 1 of the method for completing the square described in this section. Which one is it? Solve it.

12. Only one of the equations is set up so that the values of a, b, and c can be determined immediately. Which one is it? Solve it.

Solve each equation using the zero-factor property. ***See Example 1.***

13. $x^2 - 5x + 6 = 0$ **14.** $x^2 + 2x - 8 = 0$ **15.** $5x^2 - 3x - 2 = 0$

16. $2x^2 - x - 15 = 0$ **17.** $-4x^2 + x = -3$ **18.** $-6x^2 + 7x = -10$

19. $x^2 - 100 = 0$ **20.** $x^2 - 64 = 0$ **21.** $4x^2 - 4x + 1 = 0$

22. $9x^2 - 12x + 4 = 0$ **23.** $25x^2 + 30x + 9 = 0$ **24.** $36x^2 + 60x + 25 = 0$

Solve each equation using the square root property. ***See Example 2.***

25. $x^2 = 16$ **26.** $x^2 = 121$ **27.** $27 - x^2 = 0$

28. $48 - x^2 = 0$ **29.** $x^2 = -81$ **30.** $x^2 = -400$

31. $(3x - 1)^2 = 12$ **32.** $(4x + 1)^2 = 20$ **33.** $(x + 5)^2 = -3$

34. $(x - 4)^2 = -5$ **35.** $(5x - 3)^2 = -3$ **36.** $(-2x + 5)^2 = -8$

Solve each equation using completing the square. ***See Examples 3 and 4.***

37. $x^2 - 4x + 3 = 0$ **38.** $x^2 - 7x + 12 = 0$ **39.** $2x^2 - x - 28 = 0$

40. $4x^2 - 3x - 10 = 0$ **41.** $x^2 - 2x - 2 = 0$ **42.** $x^2 - 10x + 18 = 0$

43. $2x^2 + x = 10$ **44.** $3x^2 + 2x = 5$ **45.** $-2x^2 + 4x + 3 = 0$

46. $-3x^2 + 6x + 5 = 0$ **47.** $-4x^2 + 8x = 7$ **48.** $-3x^2 + 9x = 7$

Concept Check *Answer each question.*

49. Francisco claimed that the equation

$$x^2 - 8x = 0$$

cannot be solved by the quadratic formula since there is no value for c. Is he correct?

50. Francesca, Francisco's twin sister, claimed that the equation

$$x^2 - 19 = 0$$

cannot be solved by the quadratic formula since there is no value for b. Is she correct?

Solve each equation using the quadratic formula. ***See Examples 5 and 6.***

51. $x^2 - x - 1 = 0$ **52.** $x^2 - 3x - 2 = 0$ **53.** $x^2 - 6x = -7$

54. $x^2 - 4x = -1$ **55.** $x^2 = 2x - 5$ **56.** $x^2 = 2x - 10$

57. $-4x^2 = -12x + 11$ **58.** $-6x^2 = 3x + 2$ **59.** $\frac{1}{2}x^2 + \frac{1}{4}x - 3 = 0$

60. $\frac{2}{3}x^2 + \frac{1}{4}x = 3$ **61.** $0.2x^2 + 0.4x - 0.3 = 0$

62. $0.1x^2 - 0.1x = 0.3$ **63.** $(4x - 1)(x + 2) = 4x$

64. $(3x + 2)(x - 1) = 3x$ **65.** $(x - 9)(x - 1) = -16$

66. *Concept Check* Why do the following two equations have the same solution set? (Do not solve.)

$$-2x^2 + 3x - 6 = 0 \quad \text{and} \quad 2x^2 - 3x + 6 = 0$$

Solve each cubic equation using factoring and the quadratic formula. ***See Example 7.***

67. $x^3 - 8 = 0$ **68.** $x^3 - 27 = 0$

69. $x^3 + 27 = 0$ **70.** $x^3 + 64 = 0$

Solve each equation for the specified variable. (Assume no denominators are 0.) ***See Example 8.***

71. $s = \frac{1}{2}gt^2$, for t **72.** $\mathscr{A} = \pi r^2$, for r

73. $F = \frac{kMv^2}{r}$, for v **74.** $E = \frac{e^2k}{2r}$, for e

75. $r = r_0 + \frac{1}{2}at^2$, for t **76.** $s = s_0 + gt^2 + k$, for t

77. $h = -16t^2 + v_0 t + s_0$, for t **78.** $S = 2\pi rh + 2\pi r^2$, for r

For each equation, ***(a)*** *solve for x in terms of y, and* ***(b)*** *solve for y in terms of x.* ***See Example 8.***

79. $4x^2 - 2xy + 3y^2 = 2$ **80.** $3y^2 + 4xy - 9x^2 = -1$

81. $2x^2 + 4xy - 3y^2 = 2$ **82.** $5x^2 - 6xy + 2y^2 = 1$

Evaluate the discriminant for each equation. Then use it to determine the number of distinct solutions, and tell whether they are rational, irrational, *or* nonreal complex numbers. *(Do not solve the equation.)* ***See Example 9.***

83. $x^2 - 8x + 16 = 0$ **84.** $x^2 + 4x + 4 = 0$ **85.** $3x^2 + 5x + 2 = 0$

86. $8x^2 = -14x - 3$ **87.** $4x^2 = -6x + 3$ **88.** $2x^2 + 4x + 1 = 0$

89. $9x^2 + 11x + 4 = 0$ **90.** $3x^2 = 4x - 5$ **91.** $8x^2 - 72 = 0$

EXAMPLE 1 Using the Zero-Factor Property

Solve $6x^2 + 7x = 3$.

SOLUTION

$$6x^2 + 7x = 3 \quad \text{Don't factor out } x \text{ here.}$$

$$6x^2 + 7x - 3 = 0 \quad \text{Standard form}$$

$$(3x - 1)(2x + 3) = 0 \quad \text{Factor.}$$

$$3x - 1 = 0 \quad \text{or} \quad 2x + 3 = 0 \quad \text{Zero-factor property}$$

$$3x = 1 \quad \text{or} \quad 2x = -3 \quad \text{Solve each equation.}$$

$$x = \frac{1}{3} \quad \text{or} \quad x = -\frac{3}{2}$$

CHECK

$$6x^2 + 7x = 3 \quad \text{Original equation}$$

$$6\left(\frac{1}{3}\right)^2 + 7\left(\frac{1}{3}\right) \stackrel{?}{=} 3 \quad \text{Let } x = \tfrac{1}{3}. \qquad 6\left(-\frac{3}{2}\right)^2 + 7\left(-\frac{3}{2}\right) \stackrel{?}{=} 3 \quad \text{Let } x = -\tfrac{3}{2}.$$

$$\frac{6}{9} + \frac{7}{3} \stackrel{?}{=} 3 \qquad \frac{54}{4} - \frac{21}{2} \stackrel{?}{=} 3$$

$$3 = 3 \ \checkmark \ \text{True} \qquad 3 = 3 \ \checkmark \ \text{True}$$

Both values check because true statements result. The solution set is $\left\{\frac{1}{3}, -\frac{3}{2}\right\}$.

Now Try Exercise 15.

The Square Root Property When a quadratic equation can be written in the form $x^2 = k$, where k is a constant, the equation can be solved as follows.

$$x^2 = k$$

$$x^2 - k = 0 \quad \text{Subtract } k.$$

$$\left(x - \sqrt{k}\right)\left(x + \sqrt{k}\right) = 0 \quad \text{Factor.}$$

$$x - \sqrt{k} = 0 \quad \text{or} \quad x + \sqrt{k} = 0 \quad \text{Zero-factor property}$$

$$x = \sqrt{k} \quad \text{or} \quad x = -\sqrt{k} \quad \text{Solve each equation.}$$

This proves the **square root property.**

Square Root Property

If $x^2 = k$, then $\boldsymbol{x = \sqrt{k}}$ **or** $\boldsymbol{x = -\sqrt{k}}$.

That is, the solution set of $x^2 = k$ is

$$\left\{\sqrt{k}, -\sqrt{k}\right\}, \quad \textit{which may be abbreviated} \quad \left\{\pm\sqrt{k}\right\}.$$

Both solutions $\sqrt{k}$ and $-\sqrt{k}$ of $x^2 = k$ are real if $k > 0$. Both are pure imaginary if $k < 0$. If $k < 0$, then we write the solution set as

$$\left\{\pm i\sqrt{|k|}\right\}.$$

If $k = 0$, then there is only one distinct solution, 0, sometimes called a **double solution.**

EXAMPLE 2 Using the Square Root Property

Solve each quadratic equation.

(a) $x^2 = 17$ **(b)** $x^2 = -25$ **(c)** $(x-4)^2 = 12$

SOLUTION

(a)
$$x^2 = 17$$
$$x = \pm\sqrt{17} \quad \text{Square root property}$$

The solution set is $\{\pm\sqrt{17}\}$.

(b)
$$x^2 = -25$$
$$x = \pm\sqrt{-25} \quad \text{Square root property}$$
$$x = \pm 5i \quad \sqrt{-1} = i$$

The solution set is $\{\pm 5i\}$.

(c)
$$(x-4)^2 = 12$$
$$x - 4 = \pm\sqrt{12} \quad \text{Generalized square root property}$$
$$x = 4 \pm \sqrt{12} \quad \text{Add 4.}$$
$$x = 4 \pm 2\sqrt{3} \quad \sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$$

CHECK $(x-4)^2 = 12$ Original equation

$$\left(4 + 2\sqrt{3} - 4\right)^2 \stackrel{?}{=} 12 \quad \text{Let } x = 4 + 2\sqrt{3}.$$
$$\left(2\sqrt{3}\right)^2 \stackrel{?}{=} 12$$
$$2^2 \cdot \left(\sqrt{3}\right)^2 \stackrel{?}{=} 12$$
$$12 = 12 \ \checkmark \quad \text{True}$$

$$\left(4 - 2\sqrt{3} - 4\right)^2 \stackrel{?}{=} 12 \quad \text{Let } x = 4 - 2\sqrt{3}.$$
$$\left(-2\sqrt{3}\right)^2 \stackrel{?}{=} 12$$
$$(-2)^2 \cdot \left(\sqrt{3}\right)^2 \stackrel{?}{=} 12$$
$$12 = 12 \ \checkmark \quad \text{True}$$

The solution set is $\{4 \pm 2\sqrt{3}\}$.

Now Try Exercises 27, 29, and 31.

Completing the Square Any quadratic equation can be solved by the method of **completing the square,** summarized in the box below. While this method may seem tedious, it has several useful applications, including analyzing the graph of a parabola and developing a general formula for solving quadratic equations.

Solving a Quadratic Equation Using Completing the Square

To solve $ax^2 + bx + c = 0$, where $a \neq 0$, using completing the square, follow these steps.

Step 1 If $a \neq 1$, divide each side of the equation by a.

Step 2 Rewrite the equation so that the constant term is alone on one side of the equality symbol.

Step 3 Square half the coefficient of x, and add this square to each side of the equation.

Step 4 Factor the resulting trinomial as a perfect square and combine like terms on the other side.

Step 5 Use the square root property to complete the solution.

EXAMPLE 3 Using Completing the Square ($a = 1$)

Solve $x^2 - 4x - 14 = 0$.

SOLUTION $x^2 - 4x - 14 = 0$

Step 1 This step is not necessary because $a = 1$.

Step 2 $x^2 - 4x = 14$ Add 14 to each side.

Step 3 $x^2 - 4x + 4 = 14 + 4$ $\left[\frac{1}{2}(-4)\right]^2 = 4$; Add 4 to each side.

Step 4 $(x - 2)^2 = 18$ Factor. Combine like terms.

Step 5 $x - 2 = \pm\sqrt{18}$ Square root property

Take *both* roots.

$x = 2 \pm \sqrt{18}$ Add 2 to each side.

$x = 2 \pm 3\sqrt{2}$ Simplify the radical.

The solution set is $\left\{2 \pm 3\sqrt{2}\right\}$. **Now Try Exercise 41.**

EXAMPLE 4 Using Completing the Square ($a \neq 1$)

Solve $9x^2 - 12x + 9 = 0$.

SOLUTION $9x^2 - 12x + 9 = 0$

Step 1 $x^2 - \frac{4}{3}x + 1 = 0$ Divide by 9 so that $a = 1$.

Step 2 $x^2 - \frac{4}{3}x = -1$ Subtract 1 from each side.

Step 3 $x^2 - \frac{4}{3}x + \frac{4}{9} = -1 + \frac{4}{9}$ $\left[\frac{1}{2}\left(-\frac{4}{3}\right)\right]^2 = \frac{4}{9}$; Add $\frac{4}{9}$ to each side.

Step 4 $\left(x - \frac{2}{3}\right)^2 = -\frac{5}{9}$ Factor. Combine like terms.

Step 5 $x - \frac{2}{3} = \pm\sqrt{-\frac{5}{9}}$ Square root property

$x - \frac{2}{3} = \pm\frac{\sqrt{5}}{3}i$ $\sqrt{-\frac{5}{9}} = \frac{\sqrt{-5}}{\sqrt{9}} = \frac{i\sqrt{5}}{3}$, or $\frac{\sqrt{5}}{3}i$

$x = \frac{2}{3} \pm \frac{\sqrt{5}}{3}i$ Add $\frac{2}{3}$ to each side.

The solution set is $\left\{\frac{2}{3} \pm \frac{\sqrt{5}}{3}i\right\}$. **Now Try Exercise 47.**

The Quadratic Formula If we start with the equation $ax^2 + bx + c = 0$, for $a > 0$, and complete the square to solve for x in terms of the constants a, b, and c, the result is a general formula for solving any quadratic equation.

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad \text{Divide each side by } a. \text{ (Step 1)}$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \quad \text{Subtract } \tfrac{c}{a} \text{ from each side. (Step 2)}$$

Square half the coefficient of x: $\left[\frac{1}{2}\left(\frac{b}{a}\right)\right]^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$ Add $\frac{b^2}{4a^2}$ to each side. (Step 3)

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} + \frac{-c}{a}$$ Factor. Use the commutative property. (Step 4)

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} + \frac{-4ac}{4a^2}$$ Write fractions with a common denominator.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$ Add fractions.

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$ Square root property (Step 5)

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$ Since $a > 0$, $\sqrt{4a^2} = 2a$.

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$ Subtract $\frac{b}{2a}$ from each side.

Quadratic Formula This result is also true for $a < 0$. $\longrightarrow$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Combine terms on the right.

Quadratic Formula

The solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE 5 Using the Quadratic Formula (Real Solutions)

Solve $x^2 - 4x = -2$.

SOLUTION

$$x^2 - 4x + 2 = 0$$ Write in standard form. Here $a = 1$, $b = -4$, and $c = 2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic formula

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$ Substitute $a = 1$, $b = -4$, and $c = 2$.

The fraction bar extends *under* $-b$.

$$x = \frac{4 \pm \sqrt{16 - 8}}{2}$$ Simplify.

$$x = \frac{4 \pm 2\sqrt{2}}{2}$$ $\sqrt{16 - 8} = \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

$$x = \frac{2\left(2 \pm \sqrt{2}\right)}{2}$$ Factor out 2 in the numerator.

Factor first, then divide.

$$x = 2 \pm \sqrt{2}$$ Lowest terms

The solution set is $\left\{2 \pm \sqrt{2}\right\}$.

Now Try Exercise 53.

CAUTION ***Remember to extend the fraction bar in the quadratic formula under the $-b$ term in the numerator.***

Throughout this text, unless otherwise specified, we use the set of complex numbers as the domain when solving equations of degree 2 or greater.

EXAMPLE 6 Using the Quadratic Formula (Nonreal Complex Solutions)

Solve $2x^2 = x - 4$.

SOLUTION

$$2x^2 - x + 4 = 0 \quad \text{Write in standard form.}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(4)}}{2(2)} \quad \text{Quadratic formula with } a = 2, b = -1, c = 4$$

Use parentheses and substitute carefully to avoid errors.

$$x = \frac{1 \pm \sqrt{1 - 32}}{4}$$

$$x = \frac{1 \pm \sqrt{-31}}{4} \quad \text{Simplify.}$$

$$x = \frac{1 \pm i\sqrt{31}}{4} \quad \sqrt{-1} = i$$

The solution set is $\left\{\frac{1}{4} \pm \frac{\sqrt{31}}{4}i\right\}$.

Now Try Exercise 57.

The equation $x^3 + 8 = 0$ is a **cubic equation** because the greatest degree of the terms is 3. While a quadratic equation (degree 2) can have as many as two solutions, a cubic equation (degree 3) can have as many as three solutions. The maximum possible number of solutions corresponds to the degree of the equation.

EXAMPLE 7 Solving a Cubic Equation

Solve $x^3 + 8 = 0$ using factoring and the quadratic formula.

SOLUTION

$$x^3 + 8 = 0$$

$$(x + 2)(x^2 - 2x + 4) = 0 \quad \text{Factor as a sum of cubes.}$$

$$x + 2 = 0 \quad \text{or} \quad x^2 - 2x + 4 = 0 \quad \text{Zero-factor property}$$

$$x = -2 \quad \text{or} \quad x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)} \quad \text{Quadratic formula with } a = 1, b = -2, c = 4$$

$$x = \frac{2 \pm \sqrt{-12}}{2} \quad \text{Simplify.}$$

$$x = \frac{2 \pm 2i\sqrt{3}}{2} \quad \text{Simplify the radical.}$$

$$x = \frac{2(1 \pm i\sqrt{3})}{2} \quad \text{Factor out 2 in the numerator.}$$

$$x = 1 \pm i\sqrt{3} \quad \text{Divide out the common factor.}$$

The solution set is $\left\{-2, 1 \pm i\sqrt{3}\right\}$.

Now Try Exercise 67.