

1.2 Exercises

CONCEPT PREVIEW Solve each problem.

- Time Traveled** How long will it take a car to travel 400 mi at an average rate of 50 mph?
- Distance Traveled** If a train travels at 80 mph for 15 min, what is the distance traveled?
- Investing** If a person invests \$500 at 2% simple interest for 4 yr, how much interest is earned?
- Value of Coins** If a jar of coins contains 40 half-dollars and 200 quarters, what is the monetary value of the coins?
- Acid Mixture** If 120 L of an acid solution is 75% acid, how much pure acid is there in the mixture?
- Sale Price** Suppose that a computer that originally sold for x dollars has been discounted 60%. Which one of the following expressions does not represent its sale price?

A. $x - 0.60x$ B. $0.40x$ C. $\frac{4}{10}x$ D. $x - 0.60$

- Acid Mixture** Suppose two acid solutions are mixed. One is 26% acid and the other is 34% acid. Which one of the following concentrations cannot possibly be the concentration of the mixture?

A. 24% B. 30% C. 31% D. 33%

- Unknown Numbers** Consider the following problem.

The difference between seven times a number and 9 is equal to five times the sum of the number and 2. Find the number.

If x represents the number, which equation is correct for solving this problem?

A. $7x - 9 = 5(x + 2)$ B. $9 - 7x = 5(x + 2)$
C. $7x - 9 = 5x + 2$ D. $9 - 7x = 5x + 2$

- Unknown Numbers** Consider the following problem.

One number is 3 less than 6 times a second number. Their sum is 46. Find the numbers.

If x represents the second number, which equation is correct for solving this problem?

A. $46 - (x + 3) = 6x$ B. $(3 - 6x) + x = 46$
C. $46 - (3 - 6x) = x$ D. $(6x - 3) + x = 46$

- Dimensions of a Rectangle** Which one or more of the following cannot be a correct equation to solve a geometry problem, if x represents the length of a rectangle? (Hint: Solve each equation and consider the solution.)

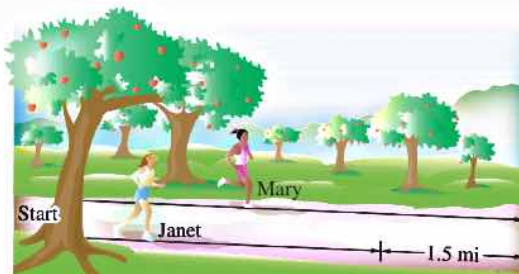
A. $2x + 2(x - 1) = 14$ B. $-2x + 7(5 - x) = 52$
C. $5(x + 2) + 5x = 10$ D. $2x + 2(x - 3) = 22$

Note: Geometry formulas can be found on the back inside cover of this book.

Solve each problem. See Example 1.

- Perimeter of a Rectangle** The perimeter of a rectangle is 294 cm. The width is 57 cm. Find the length.
- Perimeter of a Storage Shed** Michael must build a rectangular storage shed. He wants the length to be 6 ft greater than the width, and the perimeter will be 44 ft. Find the length and the width of the shed.

- 21. Distance to Work** David gets to work in 20 min when he drives his car. Riding his bike (by the same route) takes him 45 min. His average driving speed is 4.5 mph greater than his average speed on his bike. How far does he travel to work?
- 22. Speed of a Plane** Two planes leave Los Angeles at the same time. One heads south to San Diego, while the other heads north to San Francisco. The San Diego plane flies 50 mph slower than the San Francisco plane. In $\frac{1}{2}$ hr, the planes are 275 mi apart. What are their speeds?
- 23. Running Times** Mary and Janet are running in the Apple Hill Fun Run. Mary runs at 7 mph, Janet at 5 mph. If they start at the same time, how long will it be before they are 1.5 mi apart?



- 24. Running Times** If the run in Exercise 23 has a staggered start, and Janet starts first, with Mary starting 10 min later, how long will it be before Mary catches up with Janet?
- 25. Track Event Speeds** At the 2008 Summer Olympics in Beijing, China, Usain Bolt (Jamaica) set a new Olympic and world record in the 100-m dash with a time of 9.69 sec. If this pace could be maintained for an entire 26-mi marathon, what would his time be? How would this time compare to the fastest time for a marathon, which is 2 hr, 3 min, 23 sec, set in 2013? (Hint: 1 m $\approx$ 3.281 ft.) (Source: *Sports Illustrated Almanac*.)
- 26. Track Event Speeds** On August 16, 2009, at the World Track and Field Championship in Berlin, Usain Bolt set a new world record in the 100-m dash with a time of 9.58 sec. Refer to Exercise 25 and answer the questions using Bolt's 2009 time. (Source: *Sports Illustrated Almanac*.)
- 27. Boat Speed** Callie took 20 min to drive her boat upstream to water-ski at her favorite spot. Coming back later in the day, at the same boat speed, took her 15 min. If the current in that part of the river is 5 km per hr, what was her boat speed?
- 28. Wind Speed** Joe traveled against the wind in a small plane for 3 hr. The return trip with the wind took 2.8 hr. Find the speed of the wind to the nearest tenth if the speed of the plane in still air is 180 mph.

Solve each problem. See Example 3.

- 29. Acid Mixture** How many gallons of a 5% acid solution must be mixed with 5 gal of a 10% solution to obtain a 7% solution?

Strength	Gallons of Solution	Gallons of Pure Acid
5%	x	
10%	5	
7%	$x + 5$	

- 30. Acid Mixture** A student needs 10% hydrochloric acid for a chemistry experiment. How much 5% acid should she mix with 60 mL of 20% acid to get a 10% solution?

Strength	mL of Solution	mL of Pure Acid
5%	x	
20%	60	
10%	$x + 60$	

31. **Alcohol Mixture** Beau wishes to strengthen a mixture from 10% alcohol to 30% alcohol. How much pure alcohol should be added to 7 L of the 10% mixture?
32. **Alcohol Mixture** How many gallons of pure alcohol should be mixed with 20 gal of a 15% alcohol solution to obtain a mixture that is 25% alcohol?
33. **Saline Solution** How much water should be added to 8 mL of 6% saline solution to reduce the concentration to 4%?
34. **Acid Mixture** How much pure acid should be added to 18 L of 30% acid to increase the concentration to 50% acid?

Solve each problem. See Example 4.

35. **Real Estate Financing** Cody wishes to sell a piece of property for \$240,000. He wants the money to be paid off in two ways: a short-term note at 2% interest and a long-term note at 2.5%. Find the amount of each note if the total annual interest paid is \$5500.

<i>P</i> Note Amount	<i>r</i> Interest Rate (%)	<i>t</i> Time (in years)	<i>I</i> Interest Paid
x	2	1	$x(0.02)(1)$
$240,000 - x$	2.5	1	$(240,000 - x)(0.025)(1)$

36. **Buying and Selling Land** Roger bought two plots of land for a total of \$120,000. When he sold the first plot, he made a profit of 15%. When he sold the second, he lost 10%. His total profit was \$5500. How much did he pay for each piece of land?

Land Price	Rate of Return (%)	Profit (or Loss)
x	15%	
$120,000 - x$	10%	

37. **Retirement Planning** In planning her retirement, Janet deposits some money at 2.5% interest, with twice as much deposited at 3%. Find the amount deposited at each rate if the total annual interest income is \$850.
38. **Investing a Building Fund** A church building fund has invested some money in two ways: part of the money at 3% interest and four times as much at 2.75%. Find the amount invested at each rate if the total annual income from interest is \$2800.
39. **Lottery Winnings** Linda won \$200,000 in a state lottery. She first paid income tax of 30% on the winnings. She invested some of the rest at 1.5% and some at 4%, earning \$4350 interest per year. How much did she invest at each rate?
40. **Cookbook Royalties** Becky earned \$48,000 from royalties on her cookbook. She paid a 28% income tax on these royalties. The balance was invested in two ways, some of it at 3.25% interest and some at 1.75%. The investments produced \$904.80 interest per year. Find the amount invested at each rate.

(Modeling) Solve each problem. See Examples 5 and 6.

41. **Warehouse Club Membership** If the annual fee for a warehouse club membership is \$100 and the reward rate is 2% on club purchases for the year, then the linear equation

$$y = 100 - 0.02x$$

models the actual annual cost of the membership y , in dollars. Here x represents the annual amount of club purchases, also in dollars.

- (a) Determine the actual annual cost of the membership if club purchases for the year are \$2400.
- (b) What amount of club purchases would reduce the actual annual cost of the membership to \$50?
- (c) How much would a member have to spend in annual club purchases to reduce the annual membership cost to \$0?



Geometry Problems

EXAMPLE 1 Finding the Dimensions of a Square

If the length of each side of a square is increased by 3 cm, the perimeter of the new square is 40 cm more than twice the length of each side of the original square. Find the dimensions of the original square.

SOLUTION

Step 1 Read the problem. We must find the length of each side of the original square.

Step 2 Assign a variable. Since the length of a side of the original square is to be found, let the variable represent this length.

Let x = the length of a side of the original square in centimeters.

The length of a side of the new square is 3 cm more than the length of a side of the old square.

Then $x + 3$ = the length of a side of the new square.

See **Figure 1**. Now write a variable expression for the perimeter of the new square. The perimeter of a square is 4 times the length of a side.

Thus, $4(x + 3)$ = the perimeter of the new square.

Step 3 Write an equation. Translate the English sentence that follows into its equivalent algebraic equation.

$$\begin{array}{ccccccc} \text{The new} & & & \text{more} & \text{twice the length of each} \\ \text{perimeter} & \text{is} & 40 & \text{than} & \text{side of the original square.} \\ 4(x + 3) & = & 40 & + & 2x \end{array}$$

Step 4 Solve the equation.

$$4x + 12 = 40 + 2x \quad \text{Distributive property}$$

$$2x = 28 \quad \text{Subtract } 2x \text{ and } 12.$$

$$x = 14 \quad \text{Divide by 2.}$$

Step 5 State the answer. Each side of the original square measures 14 cm.

Step 6 Check. Go back to the words of the original problem to see that all necessary conditions are satisfied. The length of a side of the new square would be $14 + 3 = 17$ cm. The perimeter of the new square would be $4(17) = 68$ cm. Twice the length of a side of the original square would be $2(14) = 28$ cm. Because $40 + 28 = 68$, the answer checks.

 **Now Try Exercise 15.**

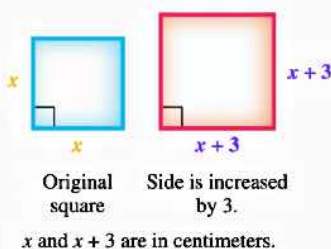


Figure 1

Motion Problems

LOOKING AHEAD TO CALCULUS

In calculus the concept of the **definite integral** is used to find the distance traveled by an object traveling at a *non-constant* velocity.

PROBLEM SOLVING HINT In a motion problem, the three components *distance*, *rate*, and *time* are denoted by the letters d , r , and t , respectively. (The *rate* is also called the *speed* or *velocity*. Here, rate is understood to be constant.) These variables are related by the following equations.

$$d = rt, \quad \text{and its related forms} \quad r = \frac{d}{t} \quad \text{and} \quad t = \frac{d}{r}$$

EXAMPLE 2 Solving a Motion Problem

Maria and Eduardo are traveling to a business conference. The trip takes 2 hr for Maria and 2.5 hr for Eduardo because he lives 40 mi farther away. Eduardo travels 5 mph faster than Maria. Find their average rates.

SOLUTION

Step 1 Read the problem. We must find Maria's and Eduardo's average rates.

Step 2 Assign a variable. Because average rates are to be found, we let the variable represent one of these rates.

Let x = Maria's rate.

Because Eduardo travels 5 mph faster than Maria, we can express his average rate using the same variable.

Then $x + 5$ = Eduardo's rate.

Make a table. The expressions in the last column were found by multiplying the corresponding rates and times.

	r	t	d
Maria	x	2	$2x$
Eduardo	$x + 5$	2.5	$2.5(x + 5)$

Summarize the given information in a table.

Use $d = rt$.

Step 3 Write an equation. Eduardo's distance traveled exceeds Maria's distance by 40 mi. Translate this into an equation.

$$\begin{array}{ccc} \text{Eduardo's} & & \text{40 more} \\ \text{distance} & \text{is} & \text{than Maria's.} \\ 2.5(x + 5) & = & 2x + 40 \end{array}$$

Step 4 Solve.

$$2.5x + 12.5 = 2x + 40 \quad \text{Distributive property}$$

$$0.5x = 27.5 \quad \text{Subtract } 2x \text{ and } 12.5.$$

$$x = 55 \quad \text{Divide by } 0.5.$$

Step 5 State the answer. Maria's rate of travel is 55 mph, and Eduardo's rate is

$$55 + 5 = 60 \text{ mph.}$$

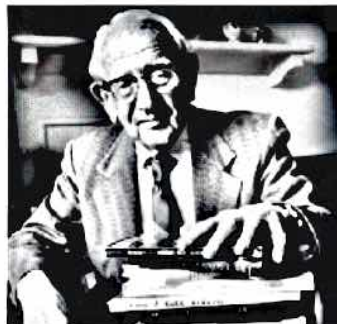
Step 6 Check. The conditions of the problem are satisfied, as shown below.

$$\text{Distance traveled by Maria: } 2(55) = 110 \text{ mi}$$

$$\text{Distance traveled by Eduardo: } 2.5(60) = 150 \text{ mi}$$

$$150 - 110 = 40 \\ \text{as required.}$$

 **Now Try Exercise 19.**



George Polya (1887–1985)

Polya, a native of Budapest, Hungary, wrote more than 250 papers and a number of books. He proposed a general outline for solving applied problems in his classic book *How to Solve It*.

Mixture Problems

Problems involving mixtures of two types of the same substance, salt solution, candy, and so on, often involve percentages.

PROBLEM-SOLVING HINT In mixture problems involving solutions,

$$\begin{array}{l} \text{rate (percent)} \\ \text{of concentration} \end{array} \cdot \text{quantity} = \begin{array}{l} \text{amount of pure} \\ \text{substance present.} \end{array}$$

The concentration of the final mixture must be between the concentrations of the two solutions making up the mixture.

EXAMPLE 3 Solving a Mixture Problem

A chemist needs a 20% solution of alcohol. She has a 15% solution on hand, as well as a 30% solution. How many liters of the 15% solution should she add to 3 L of the 30% solution to obtain the 20% solution?

SOLUTION

Step 1 Read the problem. We must find the required number of liters of 15% alcohol solution.

Step 2 Assign a variable.

Let x = the number of liters of 15% solution to be added.

Figure 2 and the table show what is happening in the problem. The numbers in the last column were found by multiplying the strengths and the numbers of liters.

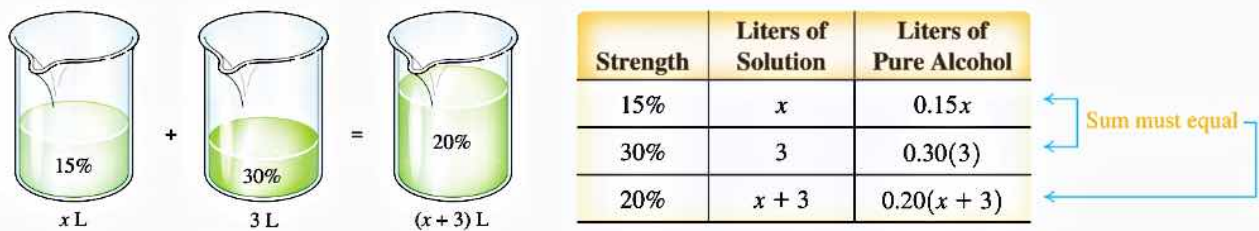


Figure 2

Step 3 Write an equation. The number of liters of pure alcohol in the 15% solution plus the number of liters in the 30% solution must equal the number of liters in the final 20% solution.

$$\begin{array}{rcl} \text{Liters of pure alcohol} & + & \text{Liters of pure alcohol} & = & \text{Liters of pure alcohol} \\ \text{in 15\% solution} & & \text{in 30\% solution} & & \text{in 20\% solution} \\ 0.15x & + & 0.30(3) & = & 0.20(x + 3) \end{array}$$

Step 4 Solve.

$$0.15x + 0.90 = 0.20x + 0.60 \quad \text{Distributive property}$$

$$0.30 = 0.05x \quad \text{Subtract 0.60 and 0.15x.}$$

$$6 = x \quad \text{Divide by 0.05.}$$

Step 5 State the answer. Thus, 6 L of 15% solution should be mixed with 3 L of 30% solution, giving $6 + 3 = 9$ L of 20% solution.

Step 6 Check. The answer checks because the amount of alcohol in the two solutions is equal to the amount of alcohol in the mixture.

$$0.15(6) + 0.9 = 0.9 + 0.9 = 1.8 \quad \text{Solutions}$$

$$0.20(6 + 3) = 0.20(9) = 1.8 \quad \text{Mixture}$$

Now Try Exercise 29.

PROBLEM-SOLVING HINT In mixed investment problems, multiply the principal amount P by the interest rate r , expressed as a decimal, and the time t , in years, to find the amount of interest earned I .

$$I = Prt \quad \text{Simple interest formula}$$

EXAMPLE 4 Solving an Investment Problem

An artist has sold a painting for \$410,000. He invests a portion of the money for 6 months at 2.65% and the rest for a year at 2.91%. His broker tells him the two investments will earn a total of \$8761. How much should be invested at each rate to obtain that amount of interest?

SOLUTION

Step 1 Read the problem. We must find the amount to be invested at each rate.

Step 2 Assign a variable.

Let x = the dollar amount to be invested for 6 months at 2.65%.

$410,000 - x$ = the dollar amount to be invested for 1 yr at 2.91%.

P Invested Amount	r Interest Rate (%)	t Time (in years)	I Interest Earned
x	2.65	0.5	$x(0.0265)(0.5)$
$410,000 - x$	2.91	1	$(410,000 - x)(0.0291)(1)$

Summarize the information in a table using the formula $I = Prt$.

Step 3 Write an equation. The sum of the two interest amounts must equal the total interest earned.

$$\begin{array}{rcccl} \text{Interest from 2.65\%} & + & \text{Interest from 2.91\%} & = & \text{Total} \\ \text{investment} & & \text{investment} & & \text{interest} \\ \hline 0.5x(0.0265) & + & 0.0291(410,000 - x) & = & 8761 \end{array}$$

Step 4 Solve. $0.01325x + 11,931 - 0.0291x = 8761$ Distributive property

$$11,931 - 0.01585x = 8761 \quad \text{Combine like terms.}$$

$$-0.01585x = -3170 \quad \text{Subtract 11,931.}$$

$$x = 200,000 \quad \text{Divide by } -0.01585.$$

Step 5 State the answer. The artist should invest \$200,000 at 2.65% for 6 months and

$$\$410,000 - \$200,000 = \$210,000$$

at 2.91% for 1 yr to earn \$8761 in interest.

Step 6 Check. The 6-month investment earns

$$\$200,000(0.0265)(0.5) = \$2650,$$

and the 1-yr investment earns

$$\$210,000(0.0291)(1) = \$6111.$$

The total amount of interest earned is

$$\$2650 + \$6111 = \$8761, \quad \text{as required.}$$

 **Now Try Exercise 35.**

Modeling with Linear Equations

A **mathematical model** is an equation (or inequality) that describes the relationship between two quantities. A **linear model** is a linear equation. The next example shows how a linear model is applied.