This midterm is 60 minutes.

Time [illegible]
Attemp[illegible]
59 M[illegible]

Question 1

1. The line x+y=1 in the 2-dimensional real vector space $\mathbb{R}^2$ is a subspace of $\mathbb{R}^2$.

- True
- False

5 pts

Question 2

2. The union of two subspaces of a vector space V is also a subspace of V.

- True
- False

5 pts

Question 3

○ False

Question 4

5 pts

4. In the vector space P(F) of polynomials over a field F, the three polynomials $x^3 + 1,\ x^3 + 2,\ x^3 + 3$ are linearly independent.

○ True

○ False

Question 5

5 pts

5. In the vector space of all real continuous functions, the three functions $1,\ \sin^2 t,\ \cos^2 t$ are linearly independent.

○ True

○ False

False

10 pts

Question 15

15. Let T be the derivative map from $P_n(F)$ to itself. Compute the matrix of T with respect to the standard basis β = $(1, x, x^2, \ldots, x^n)$.

Edit Format Table

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10 pts

Question 16

16. Let T be a linear map from $P_n\ (F)$ itself, where n> 5. Assume that the null space N(T) has dimension 3. Compute the dimension of the range R(T) as a function of n.

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p

Question 17

10 pts

17. Compute the coordinate vector $[v]_\beta$, where v is the vector x^2 in the vector space $P_2(F)$, and β is the basis $\{x, x+1, x^2+x+2\}$.

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False

Question 3

5 pts

3. The three vectors (1,1), (1,2), (1,3) in $\mathbb{R}^2$ are linearly dependent.

- ○ True
- ● False

Question 4

5 pts

4. In the vector space P(F) of polynomials over a field F, the three polynomials $x^3 + 1$, $x^3 + 2$, $x^3 + 3$ are linearly independent.

- ○ True
- ● False

Question 8

5 pts

8. Let U be a 2-dimensional subspace of a vector space V. Let W be a 3-dimensional subspace of V.

Then, the sum U + W is a 5-dimensional subspace of V.

- True
- False

Question 9

5 pts

9. The real numbers $\mathbb{R}$ is an infinite dimensional vector space over its subfield $\mathbb{Q}$ of rational numbers.

- True
- False

5 pts

Question 6

6. Let V be the subspace of $\mathbb{R}^2$ defined by the equation x+ y = 0. Let W be the subspace of $\mathbb{R}^2$ defined by the equation x-y = 0. Then, $\mathbb{R}^2 = V \oplus W$ is the direct sum of V and W.

- True
- False

5 pts

Question 7

7. Let C[-1,1] denote the real vector space of all real continuous functions on [-1,1]. Let V be the subset of all odd functions in C[-1,1], where a function f(x) is odd if f(-x)=-f(x). Then V is a subspace of C[-1,1].

- True
- False

5 pts

Question 13

13. Let $P(F_p)$ be the vector space of all polynomials over the finite field F_p of p elements. Let T: $P(F_p) \to P(F_p)$ be the derivative map. Then the null space N(T) is an infinite dimensional subspace of $P(F_p)$.

- True
- False

5 pts

Question 14

14: Let T be a linear map from a finite dimensional vector space V to itself. Assume the null space N(T) = {0}. Then, the range R(T)=V.

- True
- False

10 pts

Question 15

Question 12

5 pts

12. Let V be the vector space of all odd polynomials (odd functions) of degrees at most 10 over the field R of real numbers. Then, V is a 5-dimensional vector space over R.

- True
- False

Question 13

5 pts

13. Let $P(F_p)$ be the vector space of all polynomials over the finite field F_p of p elements. Let T: $P(F_p) \to P(F_p)$ be the derivative map. Then the null space N(T) is an infinite dimensional subspace of $P(F_p)$.

- True
- False

Question 10

5 pts

10. The map $T(x) = 2x + 3$ from the vector space $\mathbb{R}$ over $\mathbb{Q}$ to itself is a linear map.

- True
- False

Question 11

5 pts

11. Let V be a 7-dimensional vector space over a field F. Let $T(x)$ be a surjective linear map from V to itself. Then, $T(x)$ is a bijective map.

- True
- False