5. In October 2012, Apple introduced a much smaller variant of the Apple iPad, known as the iPad Mini. Weighing less than 11 ounces, it was about 50% lighter than the standard iPad. Battery tests for the iPad Mini showed a mean life of 10.25 hours (*The Wall Street Journal*, October 31, 2012). Assume that battery life of the iPad Mini is uniformly distributed between 8.5 and 12 hours.
   a. Give a mathematical expression for the probability density function of battery life.
   b. What is the probability that the battery life for an iPad Mini will be 10 hours or less?
   c. What is the probability that the battery life for an iPad Mini will be at least 11 hours?
   d. What is the probability that the battery life for an iPad Mini will be between 9.5 and 11.5 hours?
   e. In a shipment of 100 iPad Minis, how many should have a battery life of at least 9 hours?
6. A Gallup Daily Tracking Survey found that the mean daily discretionary spending by Americans earning over $90,000 per year was $136 per day (*USA Today*, July 30, 2012). The discretionary spending excluded home purchases, vehicle purchases, and regular monthly bills. Let x = the discretionary spending per day and assume that a uniform probability density function applies with $f(x) = .00625$ for $a \leq x \leq b$.
   a. Find the values of a and b for the probability density function.
   b. What is the probability that consumers in this group have daily discretionary spending between $100 and $200?
   c. What is the probability that consumers in this group have daily discretionary spending of $150 or more?
   d. What is the probability that consumers in this group have daily discretionary spending of $80 or less?
7. Suppose we are interested in bidding on a piece of land and we know one other bidder is interested.[1] The seller announced that the highest bid in excess of $10,000 will be accepted. Assume that the competitor's bid x is a random variable that is uniformly distributed between $10,000 and $15,000.
   a. Suppose you bid $12,000. What is the probability that your bid will be accepted?
   b. Suppose you bid $14,000. What is the probability that your bid will be accepted?
   c. What amount should you bid to maximize the probability that you get the property?
   d. Suppose you know someone who is willing to pay you $16,000 for the property. Would you consider bidding less than the amount in part (c)? Why or why not?

6.2 Normal Probability Distribution

Abraham de Moivre, a French mathematician, published The Doctrine *of* Chances *in 1733. He derived the normal distribution.*

The most important probability distribution for describing a continuous random variable is the **normal probability distribution**. The normal distribution has been used in a wide variety of practical applications in which the random variables are heights and weights of people, test scores, scientific measurements, amounts of rainfall, and other similar values. It is also widely used in statistical inference, which is the major topic of the remainder of this book. In such applications, the normal distribution provides a description of the likely results obtained through sampling.

Normal Curve

The form, or shape, of the normal distribution is illustrated by the bell-shaped normal curve in Figure 6.3. The probability density function that defines the bell-shaped curve of the normal distribution follows.

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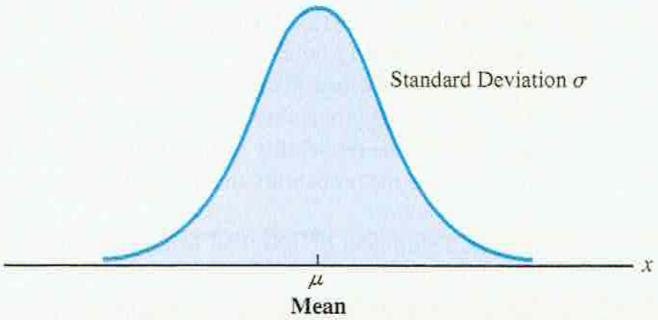
[1] This exercise is based on a problem suggested to us by Professor Roger Myerson of Northwestern University.

FIGURE 6.3 BELL-SHAPED CURVE FOR THE NORMAL DISTRIBUTION



NORMAL PROBABILITY DENSITY FUNCTION

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{6.2}$$

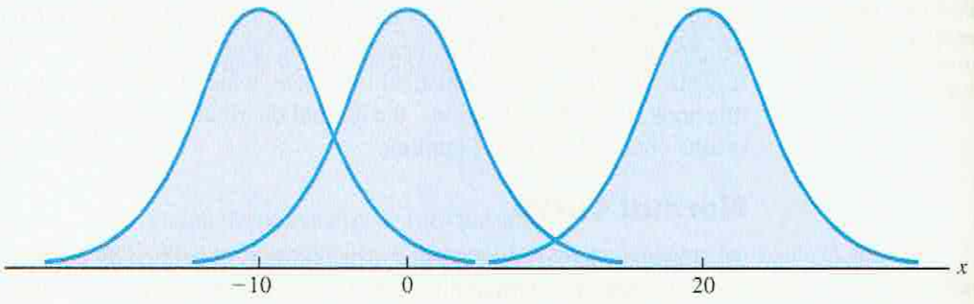
where

μ = mean
σ = standard deviation
π = 3.14159
e = 2.71828

We make several observations about the characteristics of the normal distribution.

The normal curve has two parameters, μ and σ. They determine the location and shape of the normal distribution.

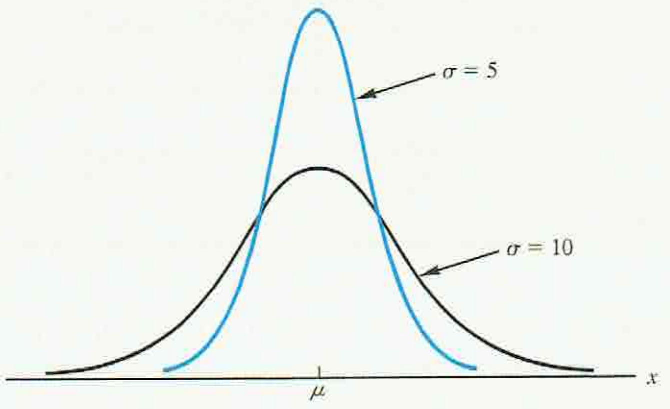
1. The entire family of normal distributions is differentiated by two parameters: the mean μ and the standard deviation σ.
2. The highest point on the normal curve is at the mean, which is also the median and mode of the distribution.
3. The mean of the distribution can be any numerical value: negative, zero, or positive. Three normal distributions with the same standard deviation but three different means (−10, 0, and 20) are shown here.



4. The normal distribution is symmetric, with the shape of the normal curve to the left of the mean a mirror image of the shape of the normal curve to the right of the mean.

The tails of the normal curve extend to infinity in both directions and theoretically never touch the horizontal axis. Because it is symmetric, the normal distribution is not skewed; its skewness measure is zero.

5. The standard deviation determines how flat and wide the normal curve is. Larger values of the standard deviation result in wider, flatter curves, showing more variability in the data. Two normal distributions with the same mean but with different standard deviations are shown here.



6. Probabilities for the normal random variable are given by areas under the normal curve. The total area under the curve for the normal distribution is 1. Because the distribution is symmetric, the area under the curve to the left of the mean is .50 and the area under the curve to the right of the mean is .50.
7. The percentage of values in some commonly used intervals are
   - **a.** 68.3% of the values of a normal random variable are within plus or minus one standard deviation of its mean;
   - **b.** 95.4% of the values of a normal random variable are within plus or minus two standard deviations of its mean;
   - **c.** 99.7% of the values of a normal random variable are within plus or minus three standard deviations of its mean.

These percentages are the basis for the empirical rule introduced in Section 3.3.

Figure 6.4 shows properties (a), (b), and (c) graphically.

Standard Normal Probability Distribution

A random variable that has a normal distribution with a mean of zero and a standard deviation of one is said to have a **standard normal probability distribution**. The letter z is commonly used to designate this particular normal random variable. Figure 6.5 is the graph of the standard normal distribution. It has the same general appearance as other normal distributions, but with the special properties of $\mu = 0$ and $\sigma = 1$.

Because $\mu = 0$ and $\sigma = 1$, the formula for the standard normal probability density function is a simpler version of equation (6.2).

STANDARD NORMAL DENSITY FUNCTION

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

FIGURE 6.4 AREAS UNDER THE CURVE FOR ANY NORMAL DISTRIBUTION

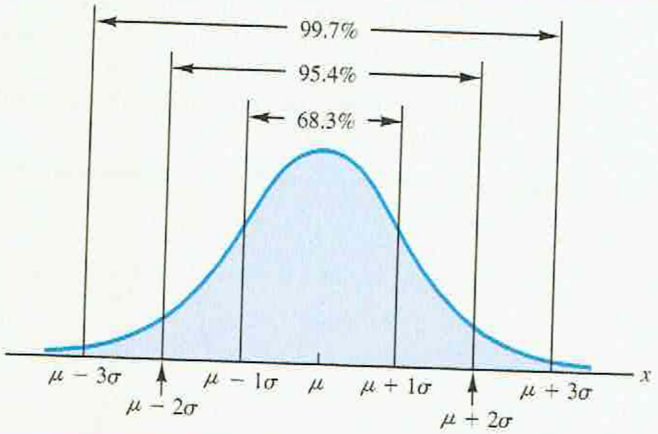
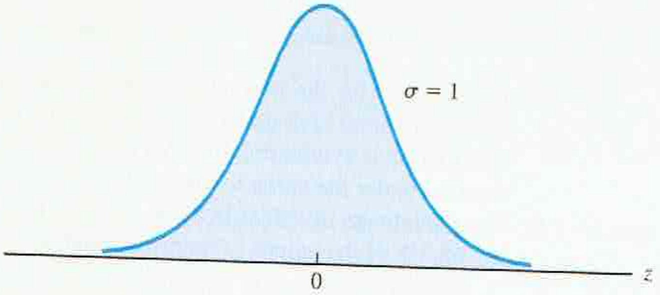


FIGURE 6.5 THE STANDARD NORMAL DISTRIBUTION



For the normal probability density function, the height of the normal curve varies and more advanced mathematics is required to compute the areas that represent probability.

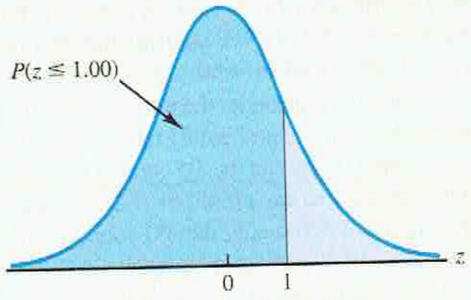
As with other continuous random variables, probability calculations with any normal distribution are made by computing areas under the graph of the probability density function. Thus, to find the probability that a normal random variable is within any specific interval, we must compute the area under the normal curve over that interval.

For the standard normal distribution, areas under the normal curve have been computed and are available in tables that can be used to compute probabilities. Such a table appears on the two pages inside the front cover of the text. The table on the left-hand page contains areas, or cumulative probabilities, for z values less than or equal to the mean of zero. The table on the right-hand page contains areas, or cumulative probabilities, for z values greater than or equal to the mean of zero.

The three types of probabilities we need to compute include (1) the probability that the standard normal random variable z will be less than or equal to a given value; (2) the probability that z will be between two given values; and (3) the probability that z will be greater than or equal to a given value. To see how the cumulative probability table for the standard normal distribution can be used to compute these three types of probabilities, let us consider some examples.

Because the standard normal random variable is continuous, $P(z \leq 1.00) = P(z < 1.00)$.

We start by showing how to compute the probability that z is less than or equal to 1.00; that is, $P(z \leq 1.00)$. This cumulative probability is the area under the normal curve to the left of $z = 1.00$ in the following graph.

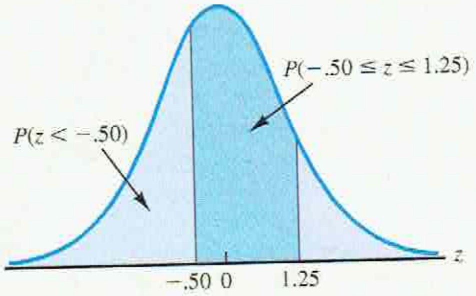


Refer to the right-hand page of the standard normal probability table inside the front cover of the text. The cumulative probability corresponding to $z = 1.00$ is the table value located at the intersection of the row labeled 1.0 and the column labeled .00. First we find 1.0 in the left column of the table and then find .00 in the top row of the table. By looking in the body of the table, we find that the 1.0 row and the .00 column intersect at the value of .8413; thus, $P(z \leq 1.00) = .8413$. The following excerpt from the probability table shows these steps.

z	**.00**	**.01**	**.02**
⋮			
.9	.8159	.8186	.8212
1.0	**.8413**	.8438	.8461
1.1	.8643	.8665	.8686
1.2	.8849	.8869	.8888
⋮			

$P(z \leq 1.00)$

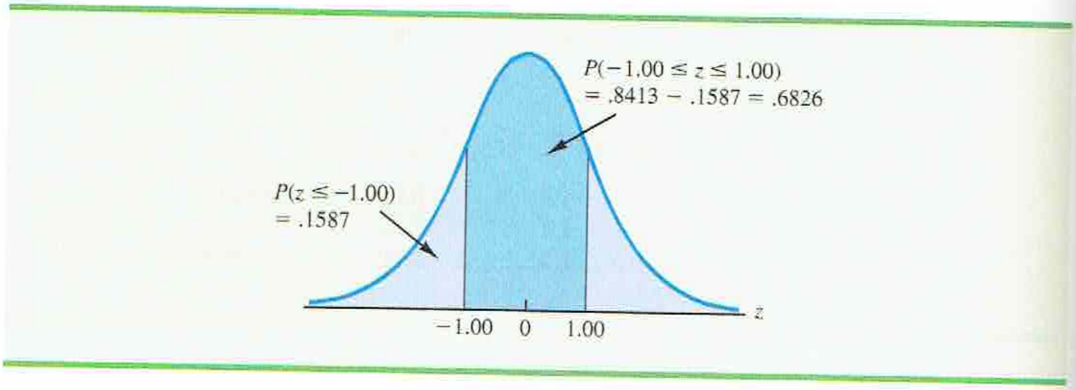
To illustrate the second type of probability calculation, we show how to compute the probability that z is in the interval between $-.50$ and 1.25; that is, $P(-.50 \leq z \leq 1.25)$. The following graph shows this area, or probability.



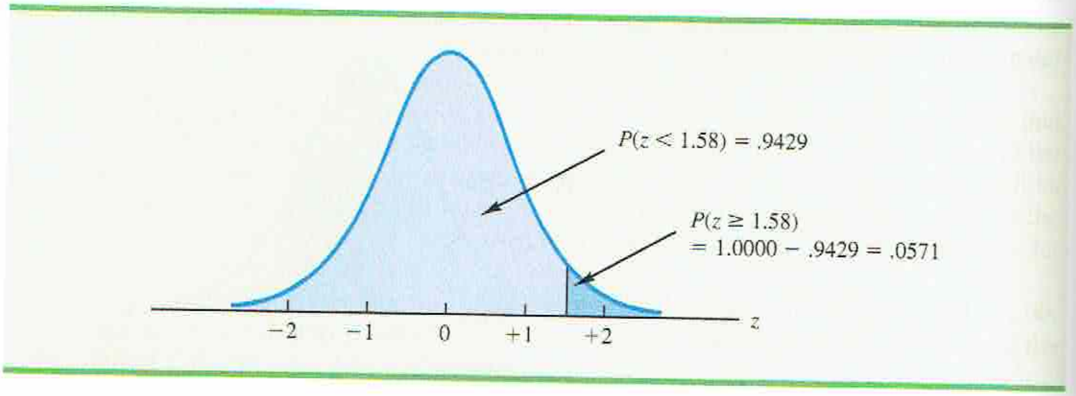
Three steps are required to compute this probability. First, we find the area under the normal curve to the left of $z = 1.25$. Second, we find the area under the normal curve to the left of $z = -.50$. Finally, we subtract the area to the left of $z = -.50$ from the area to the left of $z = 1.25$ to find $P(-.50 \leq z \leq 1.25)$.

To find the area under the normal curve to the left of $z = 1.25$, we first locate the 1.2 row in the standard normal probability table and then move across to the .05 column. Because the table value in the 1.2 row and the .05 column is .8944, $P(z \leq 1.25) = .8944$. Similarly, to find the area under the curve to the left of $z = -.50$, we use the left-hand page of the table to locate the table value in the $-.5$ row and the .00 column; with a table value of .3085, $P(z \leq -.50) = .3085$. Thus, $P(-.50 \leq z \leq 1.25) = P(z \leq 1.25) - P(z \leq -.50) = .8944 - .3085 = .5859$.

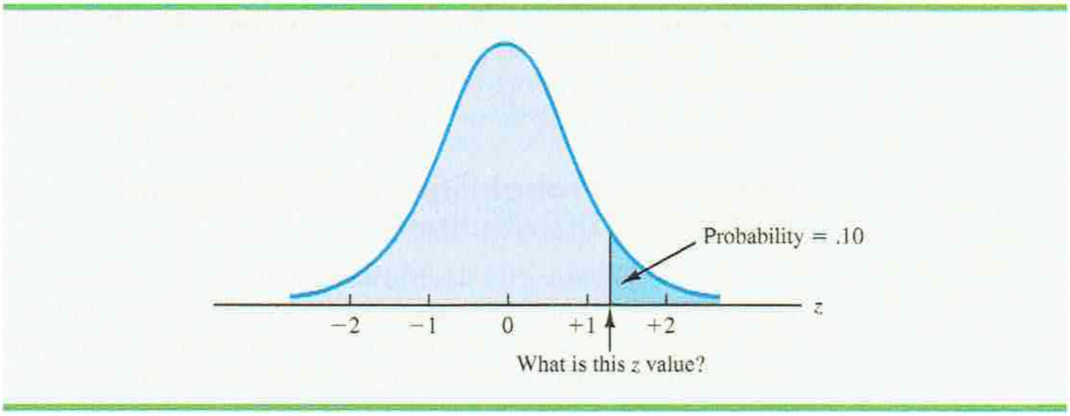
Let us consider another example of computing the probability that z is in the interval between two given values. Often it is of interest to compute the probability that a normal random variable assumes a value within a certain number of standard deviations of the mean. Suppose we want to compute the probability that the standard normal random variable is within one standard deviation of the mean; that is, $P(-1.00 \leq z \leq 1.00)$. To compute this probability we must find the area under the curve between -1.00 and 1.00. Earlier we found that $P(z \leq 1.00) = .8413$. Referring again to the table inside the front cover of the book, we find that the area under the curve to the left of $z = -1.00$ is .1587, so $P(z \leq -1.00) = .1587$. Therefore, $P(-1.00 \leq z \leq 1.00) = P(z \leq 1.00) - P(z \leq -1.00) = .8413 - .1587 = .6826$. This probability is shown graphically in the following figure.



To illustrate how to make the third type of probability computation, suppose we want to compute the probability of obtaining a z value of at least 1.58; that is, $P(z \geq 1.58)$. The value in the $z = 1.5$ row and the .08 column of the cumulative normal table is .9429; thus, $P(z < 1.58) = .9429$. However, because the total area under the normal curve is 1, $P(z \geq 1.58) = 1 - .9429 = .0571$. This probability is shown in the following figure.



In the preceding illustrations, we showed how to compute probabilities given specified z values. In some situations, we are given a probability and are interested in working backward to find the corresponding z value. Suppose we want to find a z value such that the probability of obtaining a larger z value is .10. The following figure shows this situation graphically.



Given a probability, we can use the standard normal table in an inverse fashion to find the corresponding z value.

This problem is the inverse of those in the preceding examples. Previously, we specified the z value of interest and then found the corresponding probability, or area. In this example, we are given the probability, or area, and asked to find the corresponding z value. To do so, we use the standard normal probability table somewhat differently.

Recall that the standard normal probability table gives the area under the curve to the left of a particular z value. We have been given the information that the area in the upper tail of the curve is .10. Hence, the area under the curve to the left of the unknown z value must equal .9000. Scanning the body of the table, we find that .8997 is the cumulative probability value closest to .9000. The section of the table providing this result follows.

z	**.06**	**.07**	**.08**	**.09**
.				
.				
.				
1.0	.8554	.8577	.8599	.8621
1.1	.8770	.8790	.8810	.8830
1.2	.8962	.8980	$\boxed{.8997}$	.9015
1.3	.9131	.9147	.9162	.9177
1.4	.9279	.9292	.9306	.9319
.				
.				
.				

Cumulative probability value closest to .9000

Reading the z value from the leftmost column and the top row of the table, we find that the corresponding z value is 1.28. Thus, an area of approximately .9000 (actually

.8997) will be to the left of $z = 1.28$.[2] In terms of the question originally asked, there is an approximately .10 probability of a z value larger than 1.28.

The examples illustrate that the table of cumulative probabilities for the standard normal probability distribution can be used to find probabilities associated with values of the standard normal random variable z. Two types of questions can be asked. The first type of question specifies a value, or values, for z and asks us to use the table to determine the corresponding areas or probabilities. The second type of question provides an area, or probability, and asks us to use the table to determine the corresponding z value. Thus, we need to be flexible in using the standard normal probability table to answer the desired probability question. In most cases, sketching a graph of the standard normal probability distribution and shading the appropriate area will help to visualize the situation and aid in determining the correct answer.

Computing Probabilities for Any Normal Probability Distribution

The reason for discussing the standard normal distribution so extensively is that probabilities for all normal distributions are computed by using the standard normal distribution. That is, when we have a normal distribution with any mean μ and any standard deviation σ, we answer probability questions about the distribution by first converting to the standard normal distribution. Then we can use the standard normal probability table and the appropriate z values to find the desired probabilities. The formula used to convert any normal random variable x with mean μ and standard deviation σ to the standard normal random variable z follows.

The formula for the standard normal random variable is similar to the formula we introduced in Chapter 3 for computing z-scores for a data set.

CONVERTING TO THE STANDARD NORMAL RANDOM VARIABLE

$$z = \frac{x - \mu}{\sigma} \tag{6.3}$$

A value of x equal to its mean μ results in $z = (\mu - \mu)/\sigma = 0$. Thus, we see that a value of x equal to its mean μ corresponds to $z = 0$. Now suppose that x is one standard deviation above its mean; that is, $x = \mu + \sigma$. Applying equation (6.3), we see that the corresponding z value is $z = [(\mu + \sigma) - \mu]/\sigma = \sigma/\sigma = 1$. Thus, an x value that is one standard deviation above its mean corresponds to $z = 1$. In other words, *we can interpret z as the number of standard deviations that the normal random variable x is from its mean μ.*

To see how this conversion enables us to compute probabilities for any normal distribution, suppose we have a normal distribution with $\mu = 10$ and $\sigma = 2$. What is the probability that the random variable x is between 10 and 14? Using equation (6.3), we see that at $x = 10$, $z = (x - \mu)/\sigma = (10 - 10)/2 = 0$ and that at $x = 14$, $z = (14 - 10)/2 = 4/2 = 2$. Thus, the answer to our question about the probability of x being between 10 and 14 is given by the equivalent probability that z is between 0 and 2 for the standard normal distribution. In other words, the probability that we are seeking is the probability that the random variable x is between its mean and two standard deviations above the mean. Using $z = 2.00$ and the standard normal probability table inside the front cover of the text, we see

---

[2] We could use interpolation in the body of the table to get a better approximation of the z value that corresponds to an area of .9000. Doing so to provide one more decimal place of accuracy would yield a z value of 1.282. However, in most practical situations, sufficient accuracy is obtained simply by using the table value closest to the desired probability.

that $P(z \leq 2) = .9772$. Because $P(z \leq 0) = .5000$, we can compute $P(.00 \leq z \leq 2.00) = P(z \leq 2) - P(z \leq 0) = .9772 - .5000 = .4772$. Hence the probability that x is between 10 and 14 is .4772.

Grear Tire Company Problem

We turn now to an application of the normal probability distribution. Suppose the Grear Tire Company developed a new steel-belted radial tire to be sold through a national chain of discount stores. Because the tire is a new product, Grear's managers believe that the mileage guarantee offered with the tire will be an important factor in the acceptance of the product. Before finalizing the tire mileage guarantee policy, Grear's managers want probability information about the number of miles the tires will last. Let x denote the number of miles the tire lasts.

From actual road tests with the tires, Grear's engineering group estimated that the mean tire mileage is $\mu = 36{,}500$ miles and that the standard deviation is $\sigma = 5000$. In addition, the data collected indicate that a normal distribution is a reasonable assumption. What percentage of the tires can be expected to last more than 40,000 miles? In other words, what is the probability that the tire mileage, x, will exceed 40,000? This question can be answered by finding the area of the darkly shaded region in Figure 6.6.

At $x = 40{,}000$, we have

$$z = \frac{x - \mu}{\sigma} = \frac{40{,}000 - 36{,}500}{5000} = \frac{3500}{5000} = .70$$

Refer now to the bottom of Figure 6.6. We see that a value of $x = 40{,}000$ on the Grear Tire normal distribution corresponds to a value of $z = .70$ on the standard normal distribution. Using the standard normal probability table, we see that the area under the standard normal curve to the left of $z = .70$ is .7580. Thus, $1.000 - .7580 = .2420$ is the probability that z will exceed .70 and hence x will exceed 40,000. We can conclude that about 24.2% of the tires will exceed 40,000 in mileage.

Let us now assume that Grear is considering a guarantee that will provide a discount on replacement tires if the original tires do not provide the guaranteed mileage. What should

FIGURE 6.6 GREAR TIRE COMPANY MILEAGE DISTRIBUTION

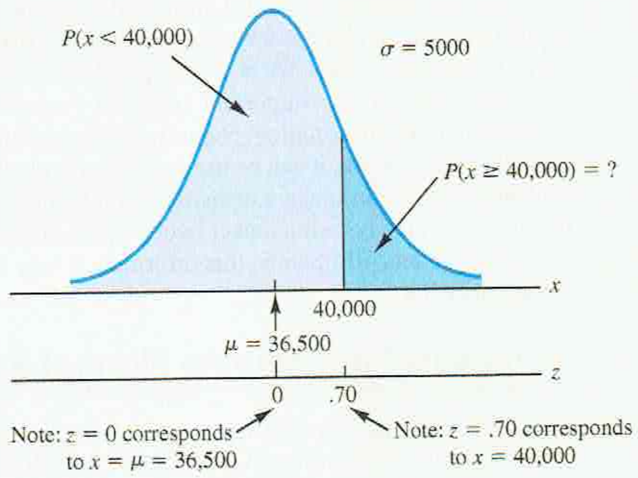
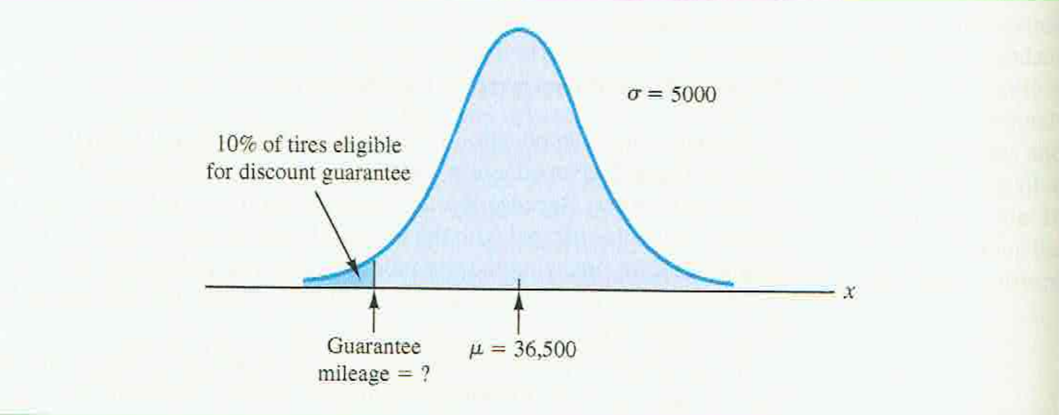


FIGURE 6.7 GREAR'S DISCOUNT GUARANTEE



the guaranteed mileage be if Grear wants no more than 10% of the tires to be eligible for the discount guarantee? This question is interpreted graphically in Figure 6.7.

According to Figure 6.7, the area under the curve to the left of the unknown guaranteed mileage must be .10. So, we must first find the z value that cuts off an area of .10 in the left tail of a standard normal distribution. Using the standard normal probability table, we see that $z = -1.28$ cuts off an area of .10 in the lower tail. Hence, $z = -1.28$ is the value of the standard normal random variable corresponding to the desired mileage guarantee on the Grear Tire normal distribution. To find the value of x corresponding to $z = -1.28$, we have

The guaranteed mileage we need to find is 1.28 standard deviations below the mean. Thus, $x = \mu - 1.28\sigma$.

$$z = \frac{x - \mu}{\sigma} = -1.28$$

$$x - \mu = -1.28\sigma$$

$$x = \mu - 1.28\sigma$$

With $\mu = 36{,}500$ and $\sigma = 5000$,

$$x = 36{,}500 - 1.28(5000) = 30{,}100$$

With the guarantee set at 30,000 miles, the actual percentage eligible for the guarantee will be 9.68%.

Thus, a guarantee of 30,100 miles will meet the requirement that approximately 10% of the tires will be eligible for the guarantee. Perhaps, with this information, the firm will set its tire mileage guarantee at 30,000 miles.

Again, we see the important role that probability distributions play in providing decision-making information. Namely, once a probability distribution is established for a particular application, it can be used to obtain probability information about the problem. Probability does not make a decision recommendation directly, but it provides information that helps the decision maker better understand the risks and uncertainties associated with the problem. Ultimately, this information may assist the decision maker in reaching a good decision.

Using Excel to Compute Normal Probabilities

Excel provides two functions for computing probabilities and z values for a standard normal probability distribution: NORM.S.DIST and NORM.S.INV. The NORM.S.DIST function computes the cumulative probability given a z value, and the NORM.S.INV function computes the z value given a cumulative probability. Two similar functions, NORM.DIST and NORM.INV, are available for computing the cumulative probability and the x value for any normal distribution. We begin by showing how to use the NORM.S.DIST and NORM.S.INV functions.

The letter S that appears in the name of the NORM.S.DIST and NORM.S.INV functions reminds us that these functions relate to the standard normal probability distribution.

The NORM.S.DIST function provides the area under the standard normal curve to the left of a given z value; thus, it provides the same cumulative probability we would obtain if we used the standard normal probability table inside the front cover of the text. Using the NORM.S.DIST function is just like having Excel look up cumulative normal probabilities for you. The NORM.S.INV function is the inverse of the NORM.S.DIST function; it takes a cumulative probability as input and provides the z value corresponding to that cumulative probability.

Let's see how both of these functions work by computing the probabilities and z values obtained earlier in this section using the standard normal probability table. Refer to Figure 6.8 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter/Access Data: Open a blank worksheet. No data are entered in the worksheet. We will simply enter the appropriate z values and probabilities directly into the formulas as needed.

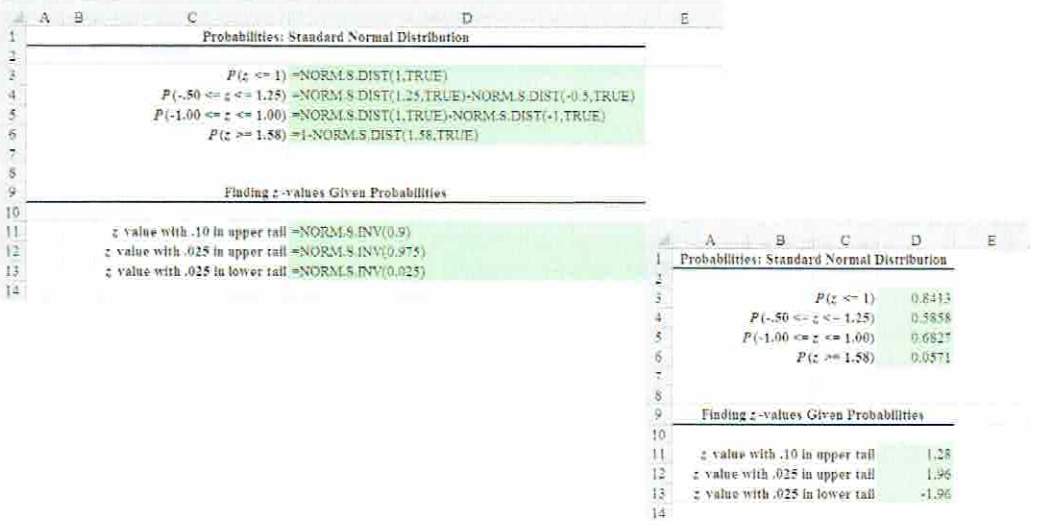
Enter Functions and Formulas: The NORM.S.DIST function has two inputs: the z value and a value of TRUE or FALSE. For the second input we enter TRUE if a cumulative probability is desired, and we enter FALSE if the height of the standard normal curve is desired. Because we will always be using NORM.S.DIST to compute cumulative probabilities, we always choose TRUE for the second input. To illustrate the use of the NORM.S.DIST function, we compute the four probabilities shown in cells D3:D6 of Figure 6.8.

The probabilities in cells D4, 0.5858, and D5, 0.6827, differ from what we computed earlier due to rounding.

To compute the cumulative probability to the left of a given z value (area in lower tail), we simply evaluate NORM.S.DIST at the z value. For instance, to compute $P(z \leq 1)$ we entered the formula =NORM.S.DIST(1,TRUE) into cell D3. The result, .8413, is the same as obtained using the standard normal probability table.

To compute the probability of z being in an interval we compute the value of NORM.S.DIST at the upper endpoint of the interval and subtract the value of NORM.S.DIST

FIGURE 6.8 EXCEL WORKSHEET FOR COMPUTING PROBABILITIES AND z VALUES FOR THE STANDARD NORMAL DISTRIBUTION



	A	B	C	D	E
1			Probabilities: Standard Normal Distribution		
2					
3			P(z <= 1)	=NORM.S.DIST(1,TRUE)	
4			P(-.50 <= z <= 1.25)	=NORM.S.DIST(1.25,TRUE)-NORM.S.DIST(-0.5,TRUE)	
5			P(-1.00 <= z <= 1.00)	=NORM.S.DIST(1,TRUE)-NORM.S.DIST(-1,TRUE)	
6			P(z >= 1.58)	=1-NORM.S.DIST(1.58,TRUE)	
7					
8					
9			Finding z-values Given Probabilities		
10					
11			z value with .10 in upper tail	=NORM.S.INV(0.9)	
12			z value with .025 in upper tail	=NORM.S.INV(0.975)	
13			z value with .025 in lower tail	=NORM.S.INV(0.025)	
14					

	A	B	C	D	E
1	Probabilities: Standard Normal Distribution				
2					
3			P(z <= 1)	0.8413	
4			P(-.50 <= z <= 1.25)	0.5858	
5			P(-1.00 <= z <= 1.00)	0.6827	
6			P(z >= 1.58)	0.0571	
7					
8					
9	Finding z-values Given Probabilities				
10					
11			z value with .10 in upper tail	1.28	
12			z value with .025 in upper tail	1.96	
13			z value with .025 in lower tail	-1.96	
14					

at the lower endpoint of the interval. For instance, to find $P(-.50 \leq z \leq 1.25)$, we entered the formula =NORM.S.DIST(1.25,TRUE)-NORM.S.DIST(-.50,TRUE) into cell D4. The interval probability in cell D5 is computed in a similar fashion.

To compute the probability to the right of a given z value (upper tail area), we must subtract the cumulative probability represented by the area under the curve below the z value (lower tail area) from 1. For example, to compute $P(z \geq 1.58)$ we entered the formula =1-NORM.S.DIST(1.58,TRUE) into cell D6.

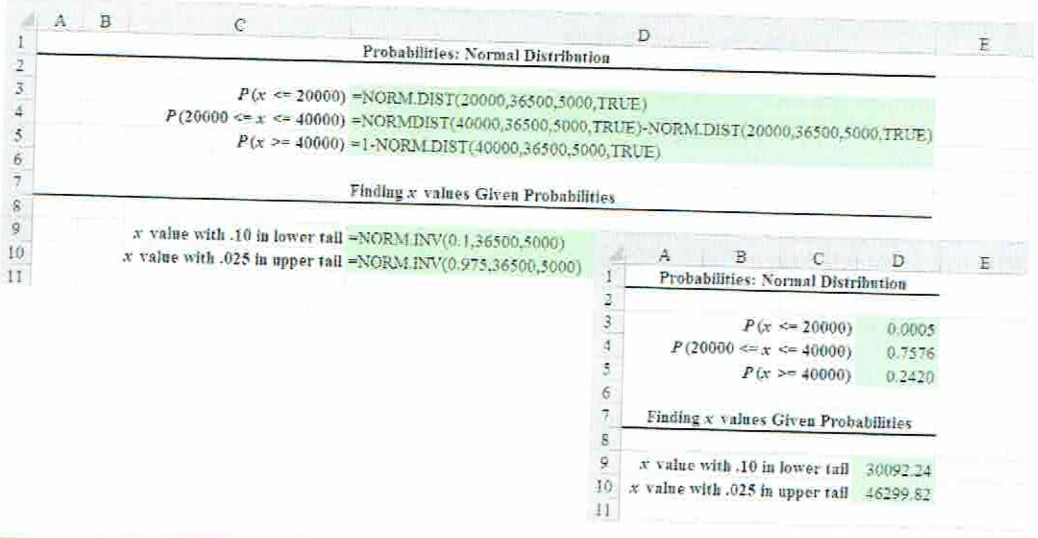
To compute the z value for a given cumulative probability (lower tail area), we use the NORM.S.INV function. To find the z value corresponding to an upper tail probability of .10, we note that the corresponding lower tail area is .90 and enter the formula =NORM.S.INV(0.9) into cell D11. Actually, NORM.S.INV(0.9) gives us the z value providing a cumulative probability (lower tail area) of .9. But it is also the z value associated with an upper tail area of .10.

Two other z values are computed in Figure 6.8. These z values will be used extensively in succeeding chapters. To compute the z value corresponding to an upper tail probability of .025, we entered the formula =NORM.S.INV(0.975) into cell D12. To compute the z value corresponding to a lower tail probability of .025, we entered the formula =NORM.S.INV(0.025) into cell D13. We see that $z = 1.96$ corresponds to an upper tail probability of .025, and $z = -1.96$ corresponds to a lower tail probability of .025.

Let us now turn to the Excel functions for computing cumulative probabilities and x values for any normal distribution. The NORM.DIST function provides the area under the normal curve to the left of a given value of the random variable x; thus it provides cumulative probabilities. The NORM.INV function is the inverse of the NORM.DIST function; it takes a cumulative probability as input and provides the value of x corresponding to that cumulative probability. The NORM.DIST and NORM.INV functions do the same thing for any normal distribution that the NORM.S.DIST and NORM.S.INV functions do for the standard normal distribution.

Let's see how both of these functions work by computing probabilities and x values for the Grear Tire Company example introduced earlier in this section. Recall that the lifetime of a Grear tire has a mean of 36,500 miles and a standard deviation of 5000 miles. Refer to Figure 6.9 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

FIGURE 6.9 EXCEL WORKSHEET FOR COMPUTING PROBABILITIES AND x VALUES FOR THE NORMAL DISTRIBUTION



	C	D
1	Probabilities: Normal Distribution	
2		
3	P(x <= 20000)	=NORM.DIST(20000,36500,5000,TRUE)
4	P(20000 <= x <= 40000)	=NORMDIST(40000,36500,5000,TRUE)-NORM.DIST(20000,36500,5000,TRUE)
5	P(x >= 40000)	=1-NORM.DIST(40000,36500,5000,TRUE)
6		
7	Finding x values Given Probabilities	
8		
9	x value with .10 in lower tail	=NORM.INV(0.1,36500,5000)
10	x value with .025 in upper tail	=NORM.INV(0.975,36500,5000)
11		

	C	D
1	Probabilities: Normal Distribution	
2		
3	P(x <= 20000)	0.0005
4	P(20000 <= x <= 40000)	0.7576
5	P(x >= 40000)	0.2420
6		
7	Finding x values Given Probabilities	
8		
9	x value with .10 in lower tail	30092.24
10	x value with .025 in upper tail	46299.82
11		

Enter/Access Data: Open a blank worksheet. No data are entered in the worksheet. We simply enter the appropriate *x* values and probabilities directly into the formulas as needed.

Enter Functions and Formulas: The NORM.DIST function has four inputs: (1) the *x* value we want to compute the cumulative probability for, (2) the mean, (3) the standard deviation, and (4) a value of TRUE or FALSE. For the fourth input, we enter TRUE if a cumulative probability is desired, and we enter FALSE if the height of the curve is desired. Because we will always be using NORM.DIST to compute cumulative probabilities, we will always choose TRUE for the fourth input.

To compute the cumulative probability to the left of a given *x* value (lower tail area), we simply evaluate NORM.DIST at the *x* value. For instance, to compute the probability that a Grear tire will last 20,000 miles or less, we entered the formula =NORM.DIST(20000,36500,5000,TRUE) into cell D3. The value worksheet shows that this cumulative probability is .0005. So, we can conclude that almost all Grear tires will last at least 20,000 miles.

To compute the probability of *x* being in an interval we compute the value of NORM.DIST at the upper endpoint of the interval and subtract the value of NORM.DIST at the lower endpoint of the interval. The formula in cell D4 provides the probability that a tire's lifetime is between 20,000 and 40,000 miles, $P(20{,}000 \le x \le 40{,}000)$. In the value worksheet, we see that this probability is .7576.

To compute the probability to the right of a given *x* value (upper tail area), we must subtract the cumulative probability represented by the area under the curve below the *x* value (lower tail area) from 1. The formula in cell D5 computes the probability that a Grear tire will last for at least 40,000 miles. We see that this probability is .2420.

To compute the *x* value for a given cumulative probability, we use the NORM.INV function. The NORM.INV function has only three inputs. The first input is the cumulative probability; the second and third inputs are the mean and standard deviation. For instance, to compute the tire mileage corresponding to a lower tail area of .1 for Grear Tire, we enter the formula =NORM.INV(0.1,36500,5000) into cell D9. From the value worksheet, we see that 10% of the Grear tires will last for 30,092.24 miles or less.

To compute the minimum tire mileage for the top 2.5% of Grear tires, we want to find the value of *x* corresponding to an area of .025 in the upper tail. This calculation is the same as finding the *x* value that provides a cumulative probability of .975. Thus we entered the formula =NORM.INV(0.975,36500,5000) into cell D10 to compute this tire mileage. From the value worksheet, we see that 2.5% of the Grear tires will last at least 46,299.82 miles.

Exercises

Methods

8. Using Figure 6.4 as a guide, sketch a normal curve for a random variable *x* that has a mean of $\mu = 100$ and a standard deviation of $\sigma = 10$. Label the horizontal axis with values of 70, 80, 90, 100, 110, 120, and 130.
9. A random variable is normally distributed with a mean of $\mu = 50$ and a standard deviation of $\sigma = 5$.
    a. Sketch a normal curve for the probability density function. Label the horizontal axis with values of 35, 40, 45, 50, 55, 60, and 65. Figure 6.4 shows that the normal curve almost touches the horizontal axis at three standard deviations below and at three standard deviations above the mean (in this case at 35 and 65).
    b. What is the probability that the random variable will assume a value between 45 and 55?
    c. What is the probability that the random variable will assume a value between 40 and 60?

10. Draw a graph for the standard normal distribution. Label the horizontal axis at values of $-3, -2, -1, 0, 1, 2$, and 3. Then compute the following probabilities.
    a. $P(z \leq 1.5)$
    b. $P(z \leq 1)$
    c. $P(1 \leq z \leq 1.5)$
    d. $P(0 < z < 2.5)$

✗ 11. Given that z is a standard normal random variable, compute the following probabilities.
    a. $P(z \leq -1.0)$
    b. $P(z \geq -1)$
    c. $P(z \geq -1.5)$
    d. $P(-2.5 \leq z)$
    e. $P(-3 < z \leq 0)$

12. Given that z is a standard normal random variable, compute the following probabilities.
    a. $P(0 \leq z \leq .83)$
    b. $P(-1.57 \leq z \leq 0)$
    c. $P(z > .44)$
    d. $P(z \geq -.23)$
    e. $P(z < 1.20)$
    f. $P(z \leq -.71)$

✓ SELF*test*

13. Given that z is a standard normal random variable, compute the following probabilities.
    a. $P(-1.98 \leq z \leq .49)$
    b. $P(.52 \leq z \leq 1.22)$
    c. $P(-1.75 \leq z \leq -1.04)$

14. Given that z is a standard normal random variable, find z for each situation.
    a. The area to the left of z is .9750.
    b. The area between 0 and z is .4750.
    c. The area to the left of z is .7291.
    d. The area to the right of z is .1314.
    e. The area to the left of z is .6700.
    f. The area to the right of z is .3300.

✓ SELF*test*

✗ 15. Given that z is a standard normal random variable, find z for each situation.
    a. The area to the left of z is .2119.
    b. The area between $-z$ and z is .9030.
    c. The area between $-z$ and z is .2052.
    d. The area to the left of z is .9948.
    e. The area to the right of z is .6915.

✗ 16. Given that z is a standard normal random variable, find z for each situation.
    a. The area to the right of z is .01.
    b. The area to the right of z is .025.
    c. The area to the right of z is .05.
    d. The area to the right of z is .10.

Applications

✗ 17. The mean cost of domestic airfares in the United States rose to an all-time high of \$385 per ticket (Bureau of Transportation Statistics website, November 2, 2012). Airfares were based on the total ticket value, which consisted of the price charged by the airlines plus any additional taxes and fees. Assume domestic airfares are normally distributed with a standard deviation of \$110.
    a. What is the probability that a domestic airfare is \$550 or more?
    b. What is the probability that a domestic airfare is \$250 or less?

c. What if the probability that a domestic airfare is between \$300 and \$500?
d. What is the cost for the highest 3% of domestic airfares?

SELF*test*

18. The average return for large-cap domestic stock funds over the three years 2009–2011 was 14.4% (*AAII Journal*, February 2012). Assume the three-year returns were normally distributed across funds with a standard deviation of 4.4%.
   a. What is the probability an individual large-cap domestic stock fund had a three-year return of at least 20%?
   b. What is the probability an individual large-cap domestic stock fund had a three-year return of 10% or less?
   c. How big does the return have to be to put a domestic stock fund in the top 10% for the three-year period?

✗ 19. Automobile repair costs continue to rise with the average cost now at \$367 per repair (*U.S. News & World Report* website, January 5, 2015). Assume that the cost for an automobile repair is normally distributed with a standard deviation of \$88. Answer the following questions about the cost of automobile repairs.
   a. What is the probability that the cost will be more than \$450?
   b. What is the probability that the cost will be less than \$250?
   c. What is the probability that the cost will be between \$250 and \$450?
   d. If the cost for your car repair is in the lower 5% of automobile repair charges, what is your cost?

20. The average price for a gallon of gasoline in the United States is \$3.73 and in Russia it is \$3.40 (*Bloomberg Businessweek*, March 5–March 11, 2012). Assume these averages are the population means in the two countries and that the probability distributions are normally distributed with a standard deviation of \$.25 in the United States and a standard deviation of \$.20 in Russia.
   a. What is the probability that a randomly selected gas station in the United States charges less than \$3.50 per gallon?
   b. What percentage of the gas stations in Russia charge less than \$3.50 per gallon?
   c. What is the probability that a randomly selected gas station in Russia charged more than the mean price in the United States?

21. A person must score in the upper 2% of the population on an IQ test to qualify for membership in Mensa, the international high-IQ society. There are 110,000 Mensa members in 100 countries throughout the world (Mensa International website, January 8, 2013). If IQ scores are normally distributed with a mean of 100 and a standard deviation of 15, what score must a person have to qualify for Mensa?

22. Television viewing reached a new high when the Nielsen Company reported a mean daily viewing time of 8.35 hours per household (*USA Today*, November 11, 2009). Use a normal probability distribution with a standard deviation of 2.5 hours to answer the following questions about daily television viewing per household.
   a. What is the probability that a household views television between 5 and 10 hours a day?
   b. How many hours of television viewing must a household have in order to be in the top 3% of all television viewing households?
   c. What is the probability that a household views television more than 3 hours a day?

23. The time needed to complete a final examination in a particular college course is normally distributed with a mean of 80 minutes and a standard deviation of 10 minutes. Answer the following questions.
   a. What is the probability of completing the exam in one hour or less?
   b. What is the probability that a student will complete the exam in more than 60 minutes but less than 75 minutes?
   c. Assume that the class has 60 students and that the examination period is 90 minutes in length. How many students do you expect will be unable to complete the exam in the allotted time?