

1. The frequency of the e.m.f. induced in the rotor of an 3 phase, 6 pole induction motor is found to have 180 cycles/min. The motor is connected to a 50 Hz 440V supply, calculate
- The speed of the motor
 - The percentage slip of the motor
 - If the full input to the rotor is 111.9kW/ph, find the rotor copper loss per phase

Ans. Given $f=50\text{Hz}$, $p = 6/2 = 3$ pole-pair

- (ii) Rotor emf frequency $f_r = 180 \text{ cycles} / 60 \text{ mins} = 3 \text{ cycle/sec} = 3 \text{ Hz}$

$$\text{By } f_r = S f, \quad S = f_r / f = 3/50 = 0.06 \Rightarrow \% \text{ Slip} = 0.06 \times 100\% = 6\%$$

- (i) Synchronous speed, $N_s = \frac{60f}{p} = \frac{60 \times 50}{3} = 1000 \text{ rev/min}$

$$\text{Speed of motor, } N_r = N_s (1 - S) = (1 - 0.06) \times 1000 = 940 \text{ rev/min}$$

- (iii) Given Rotor input power /ph, $P_{\text{rotor}} = 111.9 \text{ kW} = \boxed{3 I_2'^2 \frac{r_2'}{s}}$ (see ppt Slide 52)

$$\text{Rotor copper loss} = \boxed{3 I_2'^2 r_2'} = S \times \boxed{3 I_2'^2 \frac{r_2'}{s}} = S \times P_{\text{rotor}} = 0.06 \times 111.9 \text{ kW} = 6715 \text{ W}$$

2. A 4-pole, 3-phase induction motor is energized from a 60 Hz supply, and is running at a load condition for which the slip is 0.03. Determine
- rotor speed,
 - rotor current frequency,
 - speed of the rotor rotating field relative to the stator frame in rev/min
 - speed of the rotor magnetic field with respect to the stator rotating magnetic field, in rev/min

Ans. Given $f=60\text{Hz}$, $p = 4/2 = 2$ pole-pair, $S=0.03$

$$\text{Synchronous speed } N_s = \frac{60f}{p} = \frac{60 \times 60}{2} = 1800 \text{ rev/min}$$

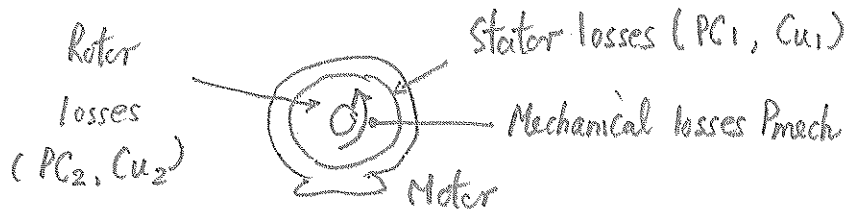
- a) Rotor speed, $N_r = N_s (1 - S) = (1 - 0.03) \times 1800 = 1764 \text{ rev/min}$

- b) Rotor current frequency, $f_r = S f = 0.03 \times 60 = 1.8 \text{ Hz}$

- c) The 4-pole stator winding also induced 4-pole magnetic field on the rotor winding. Induced current in the rotor circuit is a 3 phase current, therefore the magnetic field produced by the rotor induced current is also rotating. The speed of the rotor rotating field relative to the rotor is equal to $(N_s - N_r) = S N_s$. The rotor is rotating at a speed $N_r = (1 - S) N_s$. Therefore, the speed of the magnetic field which is rotating on the rotor surface is rotating at a speed $= S N_s + (1 - S) N_s = N_s = 1800 \text{ rev/min}$ relative to the stator winding. (i.e. Speed (not frequency) of the rotor rotating field relative to the stator frame also equal to N_s)

- d) Since the stator magnetic field is also travelling at $N_s = 1,800 \text{ rev/min}$. Therefore the stator field is travelling at the same speed of the rotor rotating field relative to the stator winding.

Relative speed between the stator field and the rotor field = 0 rpm



3. A 40 kW, 3 phase, 4 pole, 50 Hz induction motor has a full load efficiency of 85%. The friction and windage losses are one third of the no-load losses and the rotor copper loss equals the iron loss at full load. The stator resistance may be neglected and assume that the friction, windage and core losses are constant. Find the full load speed. (i.e. $N_r = ?$)

Ans. Given full load net output of the motor, $P_{out} = 40 \text{ kW}$ and efficiency = 85% = 0.85

Input power to the motor $P_{in} = P_{out}/\text{efficiency} = 40/0.85 = 47.0588 \text{ kW}$

Total losses of the motor at full load $P_{loss} = P_{in} - P_{out} = 47.0588 \times 10^3 - 40 \times 10^3 \text{ W} = 7058.8 \text{ W}$

These losses can be classified into:

- 1) Stator copper loss P_{Cu1} , this is negligible since $r_1 = 0$ (Given stator resistance neglected)
- 2) Rotor copper loss P_{Cu2} ,
- 3) Stator Core loss PC_1
- 4) Rotor Core loss, PC_2 Negligible due to rotor circuit frequency is low (only few Hz)
- 5) Mechanical losses such as friction and windage: P_{mech}

* At full load, Total loss $P_{loss} = P_{Cu1} + P_{Cu2} + PC_1 + PC_2 + P_{mech} = 7058.8 \text{ W}$

(For rotor cu loss $P_{Cu2} = 0$ at no load)

* $P_{loss} = 0 + 0 + PC_1 + 0 + P_{mech} = 7,058.8 \text{ W} \dots \dots \dots (1)$
(Note: $0 + 0$ is annotated as P_{Cu2} at full load)

From (1) No-load losses $P_{loss} = PC_1 + P_{mech} \rightarrow (PC_2 = 0 \because \text{Very low rotor circuit frequency})$

Given the friction and windage losses are one third of the no-load losses

$P_{Cu2} = 0 \because \text{Slip } S = 0, \text{ rotor circuit likes open circuit}$

* $P_{mech} = 1/3 (PC_1 + P_{mech}) \dots \dots \dots (2)$

* Given $P_{Cu2} = PC_1$ at full load (Given rotor cu equals the iron loss at full load) (3)

From (2) $P_{mech} - 1/3 P_{mech} = 1/3 PC_1 \Rightarrow PC_1 = 2 P_{mech}$

* From (3) $PC_1 = P_{Cu2} = 2 P_{mech} \Rightarrow P_{mech} = 1/2 P_{Cu2} \dots \dots \dots (4)$

Put (4) into (1) $7058.8 = P_{Cu2} + P_{Cu2} + 1/2 P_{Cu2} = 5/2 P_{Cu2} \Rightarrow P_{Cu2} = 2823.52 \text{ W}$

Since $P_{Cu2} = 3 I_2'^2 R_2'$ (see ppt slide 52)

Therefore, rotor Cu loss at full load $= 3 I_2'^2 \times R_2' = 2823.52$

Useful power
output ↓
friction & winding loss ↓

Gross mechanical power output = $P_{out} + P_{mech} = 3 I_2^2 r_2' (1-s)$ (see ppt slide 52)

s rotor
Copper loss

$$3 I_2^2 r_2' \frac{(1-s)}{s} = 40 \times 10^3 + P_{mech} = 40 \times 10^3 + 0.5 \times 2823.52 = 41411.76$$

Eqn. 4 $P_{mech} = \frac{1}{2} P_{Cu2}$

But from (5) $3 I_2^2 r_2' = 2823.52 = \text{Rotor copper loss}$

Therefore $2823.52 \frac{(1-s)}{s} = 41411.76$

Or $2823.52 - 2823.52s = 41411.76s \Rightarrow s = 0.06383$

Since synchronous Speed, $N_s = \frac{60 \times 50}{2} = 1500 \text{ rpm}$ ($N_s = \frac{\text{frequency} \times 60 \text{ seconds}}{\text{pole-pairs}}$) Slide no. 38

Rotor speed at full load, $N_r = N_s (1-s) = 1500 (1-s) = 1500 (1 - 0.06383) = 1404 \text{ rpm}$ Slide no. 37

4. A 200 kW, 4 pole, 415V, 3 phase, 50 Hz, star connected slip-ring induction motor gave the following results on test:

	Line Voltage	Line Current	Input power (3-phase)
No-load test	415	50 A	2400 W
Locked rotor test	80	150 A	6500 W

With a d.c. voltage of 10 Volts applied to any two terminals of the stator windings, the current input was found to be 100 A.

The motor is delivering full load output and connected up to a 415 Volt supply. Use the approximate equivalent circuit derived from the above test to find:

- (i) The full load current if the full load p.f. and efficiency of the motor are 0.87 and 0.92 respectively
- (ii) The ratio of starting torque to the full load torque

Ans: i) Given p.f. = 0.87, $\eta = 0.92$, $V_L = 415 \text{ V}$, $P_{out} = 200 \text{ kW}$

By $\eta = \frac{P_{out}}{P_{in}} \Rightarrow P_{in} = \frac{200 \text{ kW}}{0.92} = 217391 \text{ W}$

$P_{in} = \sqrt{3} \times V_L \times I_L \times \cos \phi$

In balanced star-connected motor, full load line current = phase current

$I_L = \frac{P_{in}}{\sqrt{3} \times V_L \times \cos \phi} = \frac{217391}{\sqrt{3} \times 415 \times 0.87} \approx 348 \angle -29.5^\circ \text{ A}$

Q 4 (ii) Ratio of starting torque to full load torque.

i.e. asking for $T = ? \rightarrow N_r = ? \rightarrow s = ? \dots$

* From no-load test, given $P_{NL} = 2400W$, $V_L = 415V$, $I_0 = 50A$

$$P_{NL} = \sqrt{3} V_L \times I_0 \times \cos \phi_0 \Rightarrow 2400 = \sqrt{3} \times 415 \times 50 \times \cos \phi_0$$

$$\phi_0 = 86.17^\circ \text{ (lagging for motor coil)}$$

$$\text{therefore } I_0 = 50 \angle -86.17^\circ = 3.34 - j49.9 \text{ A} \quad \text{--- (1)}$$

$$\text{For star-connected motor, } V_p = V_L / \sqrt{3} = 415 / \sqrt{3} = 240 \angle 0^\circ \text{ V}$$

$$\text{No-load phase current } I_0 = \frac{V_p}{Z} \Rightarrow I_0 = \frac{240}{R_0} - j \frac{240}{X_0} \quad \text{--- (2)}$$

$$\text{By equating (1) and (2)} \Rightarrow R_0 = 71.86 \Omega, \quad X_0 = 4.81 \Omega$$

core loss component

* From lock-rotor test, given $P_{LR} = 6500W$, $I_{LR} = 150A$

$$\text{By } P_{LR} = \sqrt{3} \times V_{LR} \times I_{LR} \times \cos \phi_1 \quad V_{LR} = 80V$$

$$\Rightarrow 6500 = \sqrt{3} \times 80 \times 150 \times \cos \phi_1$$

$$\Rightarrow \phi_1 = 71.78^\circ$$

If full voltage of 415V is applied to the stator winding during Lock-Rotor test, the line current will be

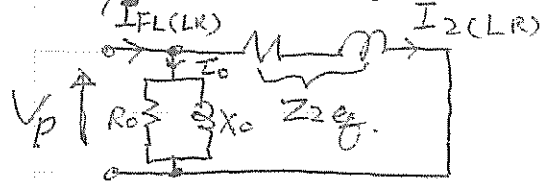
$$I_{FL(LR)} = \frac{V_L}{V_{LR}} \times I_{LR} \Rightarrow I_{FL(LR)} = \frac{415}{80} \times 150 = 778.11 \angle -71.78^\circ$$

From the motor equivalent circuit/ph (we want to find Z_{2eq} from this test)

$$I_{2(LR)} = I_{FL(LR)} - I_0$$

$$= 778.11 \angle -71.78^\circ - 50 \angle -86.17^\circ$$

$$= 729.8 \angle -70.8^\circ$$



$$Z_{2eq} = \frac{V_p}{I_{2(LR)}} = \frac{240 \angle 0^\circ}{729.8 \angle -70.8^\circ} = 0.108 + j0.311 \Omega$$

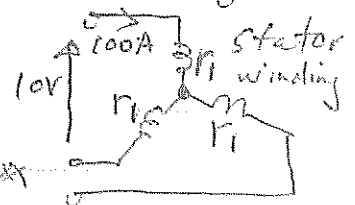
$= R_{2eq} + jX_{2eq}$

* Given when 10VDC is applied to any 2 terminals of stator winding, $I = 100A$

$$\Rightarrow 2 \times r_1 = 10V / 100A \Rightarrow r_1 = 0.05 \Omega$$

$$R_2' = R_{2eq} - r_1 = 0.108 - 0.05 = 0.058 \Omega$$

$$X_{2eq} = 0.311 \Omega$$



From (i) Full load line current, $I_{FL} = 348 \angle -29.5^\circ$ rated
act volt
(see P3)
 Rated $I_2' = I_{FL} - I_0$
 $= 348 \angle -29.5^\circ - 50 \angle -86.17^\circ \approx 323.3 \angle -22.1^\circ$

* Method (1) to find motor torque: not given in the question, answer may have error
 * Given $P_{out} = 200 \text{ kW}$ and (assume no mechanical loss)

$P_{out} = P_{gross} = 3 \times (I_2')^2 \times r_2' \left(\frac{1-s}{s} \right)$ see slide 52 power flowchart

$200 \text{ K} = 3 \times (323.3)^2 \times (0.058) \times \left(\frac{1-s}{s} \right)$

$\Rightarrow s = 0.083$ is the full load slip at rated volt

Full load speed, $N_r = N_s(1-s)$ where $N_s = \frac{60f}{P} = \frac{60 \times 50}{2}$

$N_r = 1500(1-0.083) = 1375.5 \text{ rpm}$

$T_r = \frac{P_{out}}{\omega_r} = \frac{200 \text{ K}}{\left(\frac{1375.5}{60} \right) \times 2\pi} = \frac{200 \text{ K}}{144} = 1388 \text{ Nm}$

* Method (2) to find motor torque: (Best method to get accurate ans.)

* T_r can also be found by equation in slide 61

$T_r = \frac{3P(I_2')^2 \times \left(\frac{r_2'}{s} \right)}{2\pi f_1}$ where $f_1 = 50 \text{ Hz}$
 $P = 2$

$= \frac{3 \times 2 \times (323.3)^2 \times \left(\frac{0.058}{0.083} \right)}{2\pi \times 50}$ $s = 0.083$

$= 1395 \text{ Nm}$ (very close to 1388 Nm)

* Starting torque (i.e. $s=1$ at starting)

By $T_{r|start} = \frac{3P(I_2')^2 \times \left(\frac{r_2'}{s} \right)}{2\pi f_1}$ where I_2' is the starting current found by

$T_{r|start} = \frac{3 \times 2 \times (729.8)^2 \times \left(\frac{0.058}{1} \right)}{2\pi \times 50}$ lock rotor test

$= 590 \text{ Nm}$

* Ratio of starting torque to full load torque

$= \frac{590}{1395} \approx 0.423$

5 A 22.4 kW, 400 V, three phase, 50 Hz, six-pole induction motor has a delta-connected stator (primary) winding and the following impedances, all referred to the primary, per phase:

$R_1 = 0.5 \Omega$, $R_2' = 0.39 \Omega$, total leakage reactance $X = X_1 + X_2' = 4.92 \Omega$,
No-load (magnetizing current (line)) = 10 A at 0.2 power factor.

Determine the full load line current, power factor, speed and torque, also the maximum torque and the starting torque and current. Neglect friction and windage.

By $I_2' = \frac{V}{|Z_{eq}|}$ where $Z_{eq} = R_{eq}' + jX_{eq}'$ (see slide 65)

$$I_2' = \frac{400}{\left[\left(0.5 + \frac{0.39}{s} \right)^2 + 4.92^2 \right]^{\frac{1}{2}}}$$

$R_{eq}' = r_1 + \frac{r_2'}{s} = 0.5 + \left(\frac{0.39}{s} \right) \Omega$ and $X_{eq} = 4.92 \Omega$

Output power/phase = $\frac{P_{gross}}{3}$ (see slide 52)

$$= \frac{3(I_2')^2 r_2' \left(\frac{1-s}{s} \right)}{3} = \frac{22.4 \text{ kW}}{3} = 7466 \text{ W} \quad \text{--- (1)}$$

Put I_2' into (1)

$$\frac{400^2}{\left(0.5 + \frac{0.39}{s} \right)^2 + 4.92^2} \times 0.39 \times \frac{1-s}{s} \left(\frac{s}{s} \right) = 7466$$

$$62400(s-s^2) = 7466 \left[\left(0.5s + 0.39 \right)^2 + (4.92s)^2 \right]$$

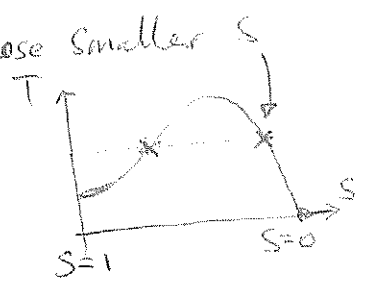
$$8.358(s-s^2) = 24.456s^2 + 0.39s + 0.152$$

$$32.814s^2 - 7.968s + 0.152 = 0$$

$s = 0.222$ or $s = 0.0208$ and we choose smaller s

Full load speed $N_r = N_s(1-s)$ where $N_s = \frac{60f}{p}$

$$= 1000(1 - 0.0208) = 979 \text{ rpm}$$



I_2' at $s = 0.0208$:

$$I_2' = \frac{400}{\left(0.5 + \frac{0.39}{0.0208} \right) + j4.92} = \frac{400}{19.25 + j4.92} = 19.84 \angle -14.13^\circ \text{ A}$$

Full load phase current $I_p = I_2' + I_0/\text{ph}$

$$I_p = 19.84 \angle -14.13 + 5.774 \angle -78.46$$

$$= 22.94 \angle -27.2^\circ$$

where no load phase current for balanced delta load

$$I_0/\text{ph} = \frac{I_L}{\sqrt{3}} = \frac{10}{\sqrt{3}} \angle \cos^{-1} 0.2 = 5.774 \angle -78.46 \text{ lagging}$$

\Rightarrow Full load line current $I_L = I_p \times \sqrt{3} = 22.94 \times \sqrt{3} = 40 \text{ A}$

overall p.f. = $\cos^{-1} 27.2 = 0.889$ lagging

Q.5 (cont.) Full load $N_r = ?$, $T_r = ?$

From above calculation, we found $s = 0.0208$ at full load.

By $N_r = N_s (1 - s) = 1000 (1 - 0.0208) = 979 \text{ rpm}$ *

By $T_r \times \omega_r = P_{out}$ Given $P_{out} = 22.4 \text{ kW}$

$$T_r = \frac{P_{out}}{\omega_r} = \frac{22.4 \text{ k}}{\left(\frac{979}{60}\right) \times 2\pi} = 218.5 \text{ Nm}$$

Full load torque *

* T_{max} occurs at $\frac{r_2'}{s} = \sqrt{r_1^2 + (X_1 + X_2')^2}$ (slide 66) *

$$\frac{0.39}{s} = \sqrt{0.5^2 + 4.92^2} \Rightarrow \frac{r_2'}{s} = 4.95$$

By $T = \frac{3 P V_1^2 \left(\frac{r_2'}{s}\right)}{2\pi f_1 [(r_1 + \frac{r_2'}{s})^2 + (X_1 + X_2')^2]}$

$$= \frac{3 \times 3 \times 400^2 \times 4.95}{2\pi \times 50 [(0.5 + 4.95)^2 + 4.92^2]} = 421 \text{ Nm}$$

* Max. Torque *

* Starting $T_{start} = ?$ Starting Current = ? Assume $P_{mech} = 0$

At starting $s = 1$, by $I_2' = \frac{V}{|Z_{eq}|}$ per phase

$$I_2' |_{start} = \frac{V}{[(r_1 + \frac{r_2'}{s})^2 + (X_1 + X_2')^2]^{\frac{1}{2}}} = \frac{400}{[(0.5 + \frac{0.39}{1})^2 + 4.92^2]^{\frac{1}{2}}}$$

or $I_2' |_{start} = \frac{400}{0.89 + j4.92} = 80 \angle -79.75^\circ$ per phase

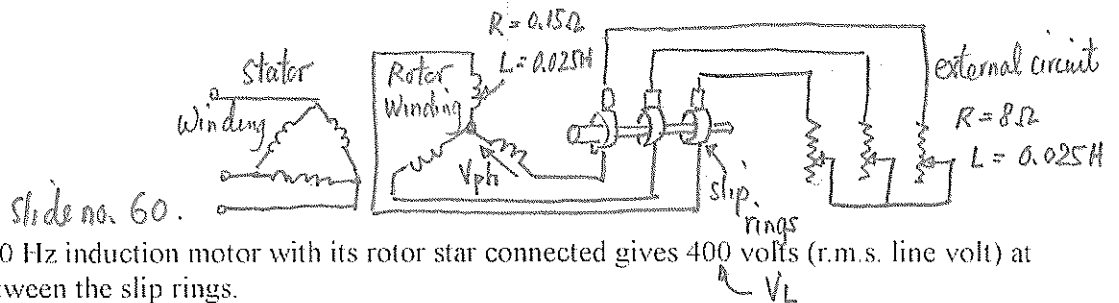
Starting line current/ph, $I_{start} = I_2' |_{start} + I_0$ (see p.6)

$$= 80 \angle -79.75^\circ + 5.744 \angle -78.46^\circ = 85.74 \angle -79.66^\circ$$

Starting line current = $85.74 \times \sqrt{3} = 148.5 \text{ A}$ *

Starting torque $T_{start} = \frac{3 P \times (I_2')^2 \times \frac{r_2'}{s}}{2\pi f_1} = \frac{3 \times 3 \times 80^2 \times \frac{0.39}{1}}{2\pi \times 50}$

$$= 71.4 \text{ Nm}$$



- 7 A 3 phase, 50 Hz induction motor with its rotor star connected gives 400 volts (r.m.s. line volt) at standstill between the slip rings.
- (i) Calculate the **current** and the **power factor** in each phase of the rotor winding at standstill when joined to a star connected external R-L circuit. Each limb has 8Ω resistor & inductance of 0.025 H. The resistance per phase of rotor winding is 0.15Ω and the inductance 0.025 H. ans. 13.05 A, 0.46
- (ii) Calculate the **current and power factor** in each rotor phase when the slip rings are short circuited and the motor is running with a slip of 4 per cent. Neglect impedance of stator winding.

Ans. (i) $I_L = ?$, $pf = ?$ at standstill (i.e. $S = 1$)

Rotor phase volt, $V_p = \frac{400}{\sqrt{3}} = 231$ V (balanced loads for 3-phase motor)

Rotor resistance per phase = $0.15 + 8 = 8.15\Omega$

Rotor reactance per phase = $2\pi f(0.025 + 0.025)$ $f = 50$ Hz
 $= 15.71\Omega$ (rotor winding + external circuit)

Rotor current/ph = $\frac{V_p}{Z_r} = \frac{231}{8.15 + j15.71}$

$= 17.7 \angle -62.58^\circ$ A. #

P.f. = $\cos^{-1} 62.58^\circ = 0.46$ lagging #

- (ii) At running condition, rotor induced emf. $E_r = SE_2$ (slide 43)
 $E_r = 0.04 \times 231 = 9.24$ V ($S = 0.04$)

When the slip ring is short circuit, $R_{ext} 8\Omega$ and $L_{ext} 0.025H$ becomes zero

New rotor resistance $R_2' = 0.15\Omega$

New rotor reactance $X_2' = SX_2 = 0.04(2\pi \times 50 \times 0.025) = 0.314\Omega$

New rotor impedance $Z_r' = 0.15 + j0.314\Omega$
 $= 0.348 \angle 64.47^\circ$

Rotor current = $\frac{E_r}{Z_r} = \frac{9.24 \angle 0^\circ}{0.348 \angle 64.47^\circ} = 26.6A \angle -64.47^\circ$

P.f. = $\cos^{-1} 64.47 = 0.43$ lagging

with slip ring short circuit:

Rotor current = 26.6A #

P.f. = 0.43 lagging #

- 8 A 6 pole, 3 phase, 50 Hz induction motor has negligible stator impedance and have rotor resistance and rotor reactance of 0.021Ω and 0.1Ω respectively per phase. What speed will give maximum torque? What external resistance is necessary to give $1/2$ maximum torque at starting?

Ans. Given $p = \frac{6}{2} = 3$, $N_s = \frac{60f}{p} = 1000 \text{ rpm}$, $S_{\max \tau} = ?$
 $r_2' = 0.021 \Omega$ $x_2' = 0.1 \Omega$ ($r_1 = 0 \Omega$, $x_1 = 0 \Omega$ * Neglect stator Z_1)

By $S_{\max \tau} = \frac{r_2'}{r_1^2 + (x_1 + x_2')^2} = \frac{r_2'}{x_2'^2} = \frac{0.021}{0.1} = 0.21$
 (see slide 66)

by $N_r = N_s (1 - S) \Rightarrow \text{Speed at max } \tau = 1000(1 - 0.21) = 790 \text{ rpm}$ #

* By $\tau_{\max} = \frac{3 \times p \times V_1^2 \times \frac{r_2'}{s}}{2\pi f_1 [(0 + \frac{r_2'}{s})^2 + (0 + x_2')^2]} = \frac{3 \times p \times V_1^2 (\frac{0.02}{0.21})}{2\pi f_1 [(\frac{0.02}{0.21})^2 + 0.1^2]}$
 (see slide 67)
 $= 4.99 \left(\frac{3 \times p \times V_1^2}{2\pi f_1} \right) - \textcircled{1}$

We want to control the torque to $0.5 \tau_{\max}$ by adding external resistor to the rotor at starting (i.e. $S=1$)

$\tau_{\text{start}} = \frac{3 \times p \times V_1^2 \times \frac{r_2'}{s}}{2\pi f_1 [(\frac{r_2'}{s})^2 + (x_2')^2]} = \frac{3 \times p \times V_1^2 \times r_2'}{2\pi f_1 [(r_2')^2 + (x_2')^2]} = 0.5 \tau_{\max}$

$\tau_{\text{start}} = \frac{3 \times p \times V_1^2 \times r_2'}{2\pi f_1 [(r_2')^2 + (0.1)^2]} = 2.495 \left(\frac{3 \times p \times V_1^2}{2\pi f_1} \right)$ (From Eqn ①)

$\Rightarrow \frac{r_2'}{(r_2')^2 + 0.01} = 2.495 \Rightarrow 2.495 r_2'^2 - r_2' + 0.02495 = 0$

$r_2' = 0.374 \Omega$ or $r_2 = 0.0267 \Omega$

Large r_2' is rejected due to high power loss is not preferred

Therefore total rotor resistance = 0.0267

additional external resistor added = $0.0267 - 0.021$
 $= 0.0057 \Omega$ #