1. A four-pole d.c. generator gives 410V on open circuit when driven at 900 rev/min. Calculate the flux per pole if the wave connected armature winding has 39 slots with 16 conductors per slot Ans: 21.9mWb

Ans. $E = 2pN\Phi Z/60a$, p = 2, a = 2 for wave winding

$$410 = \frac{2\times2\times900\times\Phi\times39\times16}{60\times2} \rightarrow \Phi = 21.9\,\text{mWb}$$

2. A six-pole d.c. generator having a lap-connected armature winding is required to give a terminal voltage of 240V when supplying an armature current of 400A. The armature has 84 slots and is driven at 700 rev/min. The resistance of the armature circuit is 0.03Ω and the useful flux per pole is about 0.03Wb. Calculate the number of conductors per slot and the actual value of the useful flux per pole. Ans: 8, 32.1mWb

Ans. For a generator, $V = E - I_aR_a$, $E = 240 + 400 \times 0.03 = 252$ Volts
$E = 2pN\Phi Z/60a$, p = 3, a = 2p = 6 for lap winding,
let Y = number of conductors in each slot,

$$252 = \frac{2\times p\times700\times0.03\times84\times Y}{60\times2p} \rightarrow Y = 8.57$$

If we take **8 conductor per pole** as the round up number, the useful flux per pole must increase to 0.03 x 8.57/8 = **0.032 Wb.**

3. A 120V, dc shunt motor draws a **total** of **8A** (line current) and drives a **constant load torque** at 840 rpm with field current = **1A**. The armature resistance of the motor is 0.8 Ω. Neglecting rotational losses, find the speed, line current, mechanical power output and efficiency of the motor when the field current is reduced to **0.8 A**. Ans: 1037 r.p.m., 9.55A, 988.75W, 86.3%

Ans. $E = k\,N\Phi \ldots(1)$, (See Powerpoint Slide 65)
$T = k\,I_a\,\Phi \ldots(2)$

For a motor, $V = E + I_aR_a$ ..(3) $\rightarrow V = E_1 + I_{a1}R_{a1}$ ..(3a) and $V = E_2 + I_{a2}R_{a1}$ …(3b)
T = constant …(5) **(Given constant load torque)**
Given $V_1 = 120$ Volts, $N_1 = 840$ rev/min , $R_a = 0.8\ \Omega$,

For the shunt motor, given **total current $I_1 = 8A$, $I_{f1} = 1A$,** => $I_{a1} = I_1 - I_{f1} = 8 - 1 = 7A$

From (3a) $120 = E_1 + 7 \times 0.8 \rightarrow E_1 = 114.4$ Volts

Given constant load torque, put (5) into (2) $I_a\Phi$ = constant i.e. $I_{a1}\Phi_1 = I_{a2}\Phi_2$ or $\frac{\Phi_1}{\Phi_2} = \frac{I_{a2}}{I_{a1}}$

and If reduced to 0.8A:

By $\Phi = kI_f$ (See ppt. Slide 69) => $\frac{\Phi_1}{\Phi_2} = \frac{I_{a2}}{I_{a1}} = \frac{I_{f1}}{I_{f2}}$ => $\frac{1}{0.8} = \frac{I_{a2}}{7} \rightarrow I_{a2} = 8.75A$,

=> $I_2 = I_{a2} + I_{f2} = 8.75 + 0.8 =$ **9.55 A (Line current ans.)**

From (3b) $V = E_2 + I_{a2}R_{a1}$ => $120 = E_2 + 8.75 \times 0.8 \rightarrow$ **$E_2 = 113$ V**

By $\Phi = kI_f$ again, given $I_{f1} = 1A$, $I_{f2} = 0.8A$ => $\frac{\Phi_1}{\Phi_2} = \frac{I_{f1}}{I_{f2}} \rightarrow \frac{\Phi_1}{\Phi_2} = \frac{1}{0.8}$

From (1) $\frac{E_1}{E_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{\Phi_1}{\Phi_2}\right) \rightarrow \frac{114.4}{113} = \left(\frac{840}{N_2}\right)\left(\frac{1}{0.8}\right) \rightarrow N_2 = 1037.15$ **rpm**

(Assume no rotational losses in the rotor) i.e. Pmechanical output = Pelectrical input = E I_a

Gross mechanical power output = E I_a = 113 x 8.75 = **988.75 W**

Efficiency of motor=$\frac{Output}{Input} \times 100\% = \frac{988.75}{120 \times (8.75 + 0.8)} \times 100\% = 86.28\%$ (no rotational loss, why η< 100% ?)

4. A separately excited dc motor is rated at 200V, 840rpm, 60A. The armature resistance is 0.08Ω. Speed control is achieved by armature voltage control for speed below the rated speed (with full field); and field control for speed above the rated speed (with rated armature voltage). Neglect rotational losses of the motor. Ans: a) 121 Volts bi) 83.6%, bii) 82.5%

a) Find the armature input voltage when the speed is 500rpm (load torque is constant) and;
b) Calculate the percentage of the full field flux required to reach 1000 rpm when the load torque is :
   i) independent of motor speed i.e. motor **torque is constant**; and
   ii) proportional to the square of motor speed.

Ans (a). Speed control by armature voltage control for speed below the rated speed (with full field)

$E = k N\Phi \cdots(1)$, $\Gamma = k\, I_a\, \Phi \cdots(2)$ (See Powerpoint Slide 65)

For a motor, $V = E + I_aR_a \cdots(3) \rightarrow V = E_1 + I_{a1}R_{a1} \ldots(3a)$ and $V_2 = E_2 + I_{a2}R_{a1} \ldots(3b)$

Given $V_1 = 200$, $N_1 = 840$, $N_2 = 500$, $I_{a1} = 60A$, $R_a = 0.08\Omega$

From (3a) => $200 = E_1 + 60 \times 0.08 \rightarrow E_1 = 195.2$ Volts

Given Φ = constant ...(5) (i.e. at fixed full field voltage, Φ = constant)

By $E = kN\Phi$ => $\frac{E_1}{E_2} = \frac{N_1\Phi}{N_2\Phi} = \frac{E_1}{E_2} = \frac{N_1}{N_2} \Rightarrow \frac{195.2}{E_2} = \frac{840}{500} \Rightarrow E_2 = 116.19V$

$V_2 = 116.9 + 60 \times 0.08 =$ **121 V** **(i.e speed control by adjusting V to reduce armature voltage E)**

→ V=200V → V=121V

bi) Speed control by field current control for speed above the rated speed (with rated armature voltage)

Given $V_1 = 200$ Volts, $I_{a1} = 60$ A, $N_1 = 840$, $N_2 = 1000$, Γ = constant, $\frac{\Phi_2}{\Phi_1} = ?\%$

i.e. E keeps at ≃195V

$E = k N\Phi$, $\frac{E_1}{E_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{\Phi_1}{\Phi_2}\right) \cdots(1)$

$\Gamma = k\, I_a\Phi \cdots(2)$, $V = E + I_aR_a \cdots(3)$

$\rightarrow V = E_1 + I_{a1}R_{a1} \ldots(3a)$ or $200 = E_1 + 60 \times 0.08$, $E_1 = 195.2$ Volts

Given: Γ = constant ...(4)

Put (4) into (2) gives $k = I_a\Phi \rightarrow I_{a1}\Phi_1 = I_{a2}\Phi_2$ or $I_{a2} = 60\frac{\Phi_1}{\Phi_2}$

From (1) $\frac{195.2}{E_2} = \left(\frac{840}{1000}\right)\left(\frac{\Phi_1}{\Phi_2}\right) \Rightarrow E_2 = 232.381\left(\frac{\Phi_2}{\Phi_1}\right)$ → (* From P.3 ans. $E_2 = 232.381 \times 0.836$ $= 194.5V$ $\simeq 195V$)

Substitute into (3b) gives: $200 = 232.381\left(\frac{\Phi_2}{\Phi_1}\right) + 60\left(\frac{\Phi_1}{\Phi_2}\right) \times 0.08$

Multiply by $\left(\frac{\Phi_2}{\Phi_1}\right)$ gives: $200\left(\frac{\Phi_2}{\Phi_1}\right) = 232.381\left(\frac{\Phi_2}{\Phi_1}\right)^2 + 4.8$

Rearranging gives: $232.381\left(\frac{\Phi_2}{\Phi_1}\right)^2 - 200\left(\frac{\Phi_2}{\Phi_1}\right) + 4.8 = 0$

On solving gives $\left(\frac{\Phi 2}{\Phi 1}\right) = 0.836$ (Ans.) or $\left(\frac{\Phi 2}{\Phi 1}\right) = 0.0247$ (i.e. reduced to 2.47% of full field)

To maintain constant Γ, armature current must increase. By $I_{a2} = 60\frac{\Phi_1}{\Phi_2} = \left[I_{a2} = 60\left(\frac{\Phi_1}{\Phi_2}\right) = 2{,}429\text{A}\right]$

armature current is too large and will burn the armature winding. So $\frac{\Phi 2}{\Phi 1}$=2.47% is rejected

bii) Given $V_1 = 200$ V, $I_{a1} = 60$ A, $N_1 = 840$, $N_2 = 1000$, Torque is proportional to the **square** of motor speed $\frac{\Phi 2}{\Phi 1} = ?$

$E = kN\Phi$, $\frac{E_1}{E_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{\Phi_1}{\Phi_2}\right)$ $\cdots(1)$ and

$\Gamma = k\, I_a \Phi \cdots (2)$, and given $\Gamma = kN^2 \ldots (4)$ (given proportional to the square of motor speed)

$\rightarrow V = E_1 + I_{a1}R_{a1}$ ... (3a) or $200 = E_1 + 60 \times 0.08$ => $E_1 = 195.2$ Volts

and $V = E_2 + I_{a2}R_{a1}$ ...(3b)

Put (4) into (2) $\rightarrow kN^2 = kI_a\Phi \rightarrow \left(\frac{N_1}{N_2}\right)^2 = \left(\frac{I_{a1}}{I_{a2}}\right)\left(\frac{\Phi_1}{\Phi_2}\right)$ or $\left(\frac{840}{1000}\right)^2 = \left(\frac{60}{I_{a2}}\right)\left(\frac{\Phi_1}{\Phi_2}\right)$

$I_{a2} = 60\left(\frac{1000}{840}\right)^2\left(\frac{\Phi 1}{\Phi 2}\right)$ or $I_{a2} = 85.034\left(\frac{\Phi 1}{\Phi 2}\right)$

From (1) $\frac{E_1}{E_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{\Phi_1}{\Phi_2}\right) \Rightarrow \frac{195.2}{E_2} = \left(\frac{840}{1000}\right)\left(\frac{\Phi_1}{\Phi_2}\right) \Rightarrow E_2 = 232.381\left(\frac{\Phi_2}{\Phi_1}\right)$ + (* From below ans. $E_2 = 232.381 \times 0.825$ $= 191.7\text{V}$ $\approx 195\text{ V}$

By $V = E_2 + I_{a2}R_{a1}$ => $200 = 232.381\left(\frac{\Phi_2}{\Phi_1}\right) + 85.034\left(\frac{\Phi_1}{\Phi_2}\right) \times 0.08$

Simplification gives $200 = 232.381\left(\frac{\Phi_2}{\Phi_1}\right) + 6.8027\left(\frac{\Phi_1}{\Phi_2}\right)$

Multiply by $\left(\frac{\Phi_2}{\Phi_1}\right)$ gives: $200\left(\frac{\Phi_2}{\Phi_1}\right) = 232.381\left(\frac{\Phi_2}{\Phi_1}\right)^2 + 6.8027$

$232.381\left(\frac{\Phi_2}{\Phi_1}\right)^2 - 200\left(\frac{\Phi_2}{\Phi_1}\right) + 6.8027 = 0$ On solving gives $\left(\frac{\Phi_2}{\Phi_1}\right) = 0.825$ or $\left(\frac{\Phi_2}{\Phi_1}\right) = 0.0355$

(i.e. reduced to 82.5% or 3.55% of full field)

Again, as shown in b(i) $\left(\frac{\Phi_2}{\Phi_1}\right) = 0.0355$ is rejected because the flux is too small which will cause the armature current to be very large $\left[I_{a2} = 85.034\left(\frac{\Phi_1}{\Phi_2}\right) = 2{,}395\text{A}\right]$ & burn the armature winding

5. A d.c. series motor, connected to 440 Volt supply, runs at 600 rev/min when taking an armature current of 50 A. Calculate the value of additional resistor which when inserted in series with the motor, will reduce the speed to 400 rev/min, the gross torque being then half its previous value. Resistance of the armature circuit = 0.2 Ω. Assume the flux to be proportional to the field current.

Ans. $E = k\,N\,\Phi \cdots(1)$, ($N=\omega_m$ in ppt)

$\Gamma = k\,I_a\,\Phi \cdots(2)$

For a motor, $V = E + I_a R_a \cdots(3)$

$\rightarrow V = E_1 + I_{a1}R_a \;\ldots(3a)$ and $V = E_2 + I_{a2}(R_a + R_{add}) \ldots(3b)$

$\Phi = kI_a$ for a **series motor** $\cdots(4)$ (i.e. $\Phi \,\alpha\, I_f$ and $I_f = I_a$ for series motor, therefore $\Gamma = kI_a^2$)

$\Gamma_2 = 0.5\,\Gamma_1$ (given) $\cdots(5)$

Given $V = 440$ Volts, $N_1 = 600$, $N_2 = 400$, $I_{a1} = 50A$, $R_a = 0.2\Omega$

From (3a) $440 = E_1 + 50 \times 0.2 \rightarrow E_1 = 430$ Volts

Substitute (4) into (1) gives $E = k\,N\,I_a \ldots(6)$

Substitute (4) into (2) gives $\Gamma = kI_a^2$ or $\frac{\Gamma_1}{\Gamma_2} = \left(\frac{I_{a1}}{I_{a2}}\right)^2 \Rightarrow \frac{1}{0.5} = \left(\frac{50}{I_{a2}}\right)^2 \Rightarrow I_{a2} = 35.3553A$

From (6) $\frac{E_1}{E_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{I_{a_1}}{I_{a_2}}\right) \Rightarrow \frac{430}{E_2} = \frac{600}{400} \times \frac{50}{35.3553} \Rightarrow E_2 = 202.7$ Volts

Substitute into (3b) gives $440 = 202.7 + 35.3553 \times (0.2 + R_{add}) \rightarrow$ Radd = 6.51Ω

6. A shunt motor runs on no load at 800 rev/min from a 240 V supply with no external resistor in the field circuit. The armature current is 2A. Calculate the resistance required in series with the shunt winding so that the motor may run at 950 rev/min when taking an armature current of 30A. Shunt winding resistance = 160Ω; armature resistance = 0.4 Ω. Assume that the flux is proportional to the field current. Ans: 39.33Ω

Ans. $E = k\,N\,\Phi \cdots(1)$,

For a motor, $V = E + I_a R_a \cdots(2)$

$\rightarrow V = E_1 + I_{a1}R_a \;\ldots(2a)$ and $V = E_2 + I_{a2}\,R_a \ldots(2b)$

Given $V = 240$ Volts, $N_1 = 800$, $N_2 = 950$, $I_{a1} = 2A$, $I_{a2} = 30A$, $R_a = 0.4\ \Omega$

(2a) gives $240 = E_1 + 2 \times 0.4 \Rightarrow E_1 = 239.2$ Volts

From (1) $\frac{E_1}{E_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{\Phi_1}{\Phi_2}\right) \Rightarrow \frac{239.2}{E_2} = \frac{800}{950}\left(\frac{\Phi_1}{\Phi_2}\right) \Rightarrow E_2 = 284.05\left(\frac{\Phi_2}{\Phi_1}\right)$

Substitute into (2b) gives $240 = 284.05\left(\frac{\Phi_2}{\Phi_1}\right) + 30 \times 0.4 \Rightarrow \left(\frac{\Phi_2}{\Phi_1}\right) = 0.8027$

This reduces the flux to 0.8027 times of its original value.

By $I_f \,\alpha\, \Phi_f \,\alpha \left(\frac{1}{R_f}\right)$ => $\left(\frac{R_{f1}}{R_{f2}}\right) = \left(\frac{\Phi_2}{\Phi_1}\right)$ Given **shunt** winding resistance $Rf_1 = 160\Omega$,

$Rf_2 = Rf_1/0.8027$ => The resistance must be increased to $Rf_2 = 160 / 0.8027 = 199.33\ \Omega$,
Additional resistance to be inserted into the field circuit = 199.33 − 160 = **39.33 Ω.**